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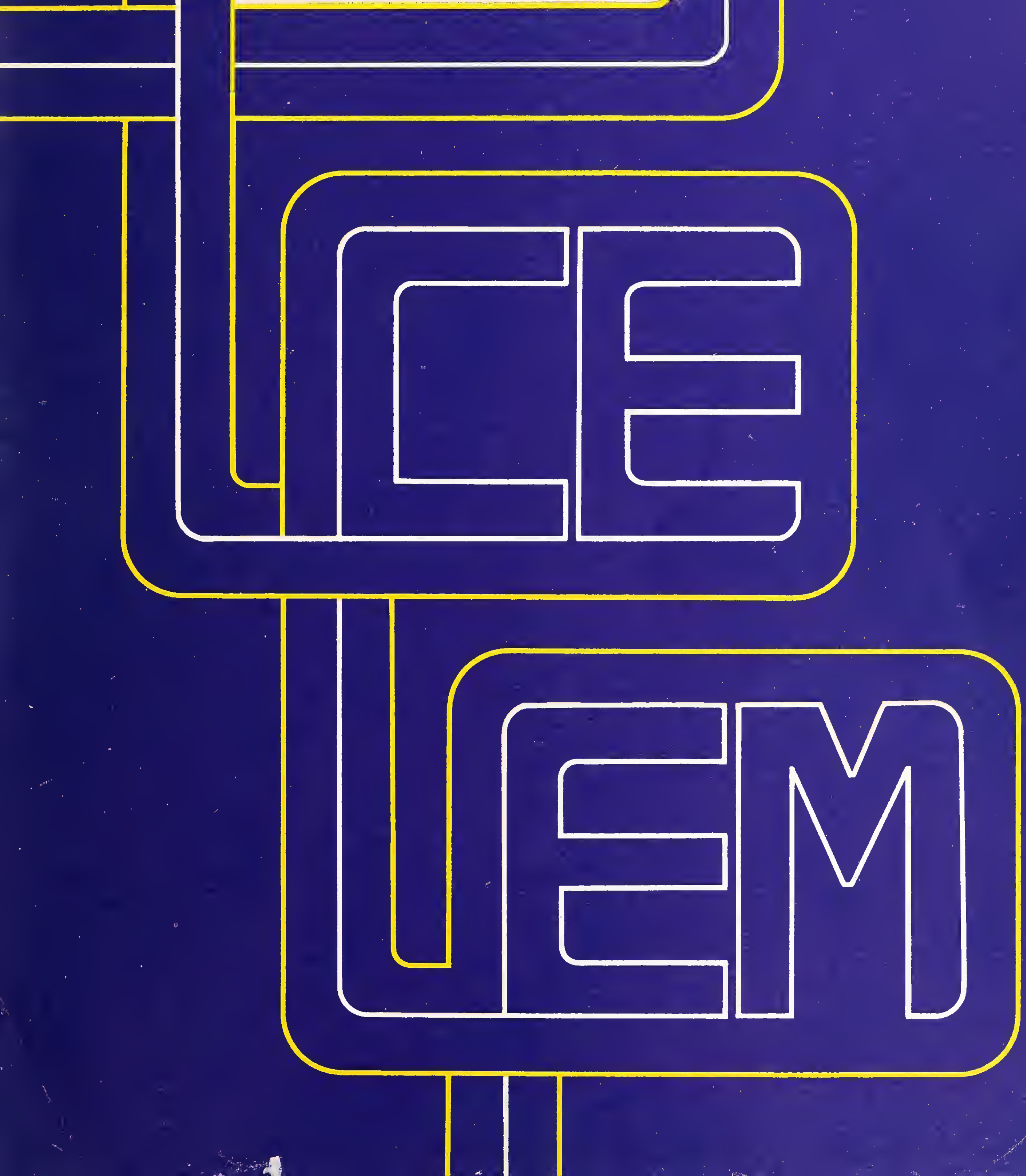

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APPLICATION HYDROLOGIC AND HYDRAULIC RESEARCH TO CULVERT SELECTION IN MONTANA

VOL I REPORT

Vol I Report
Vol II Appendices

Prepared for

Planning and Research Bureau
Department of Highways
State of Montana
Helena, Montana

in cooperation with

U.S. Department of Transportation
Federal Highway Administration

The opinions, findings and conclusions expressed in this publication are those of the authors and not necessarily those of the Montana Department of Highways or the Federal Highway Administration

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16. Abstract

A regional frequency analysis was performed, involving both natural and synthetic streamflow records, to estimate flood magnitude of various recurrence intervals for each of 230 Montana Watersheds. Using regression analysis, flood peak prediction equations for the 2, 5, 10, 25 and 50 year recurrence intervals were developed for each of 9 contiguous geographic regions in Montana. These equations predict the flood peak as the product of powers of various hydrologic watershed parameters.

A detailed discussion of the hydraulic conditions which occur for various types of culvert flow is presented along with modern criteria and methods for culvert selection.

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Chapter I

INTRODUCTION AND PREVIOUS WORK

INTRODUCTION

The objective of this study was to provide the basis for a comprehensive engineering report which would present (1) recommended methods for predicting the magnitude and frequency of occurrence of floods from small drainage areas in Montana for culvert design practice by the Montana Highway Department, (2) recommended methods for analysing the hydraulic performance of culverts, and (3) recommendations for necessary field data for satisfactory culvert design. Specifically it was requested that hydrologic methods be developed to enable flood predictions to be made for drainage areas from 0.1 to 200 sq miles located anywhere in the State.

PREVIOUS WORK

This discussion will be confined to previous work done for the purpose of engineering application in predicting of floods from Montana drainages.

U.S. GEOLOGICAL SURVEY REPORTS

V. K. Berwick[1] in 1958 presented a method for determining the magnitude and frequency of floods from drainage areas of 100 to 3000 sq mi for that portion of eastern Montana averaging roughly east of 109° longitude.

A composite regional frequency curve was developed for this area of about 60,000 sq mi by combining the flood data for 16 watersheds ranging from 38.4 to 3070. sq mi in size, expressing all floods as flood ratios by dividing the flow by the mean annual flood, developed separately for each watershed based on the Gumbel extreme value probability distribution. A homogeneity test was made to determine that the 16 watersheds could be considered hydrologically homogeneous. Median flood ratios for the 16 stations for each recurrence interval were plotted to define the composite curve which defined the ratio flood magnitude/mean annual flood vs recurrence interval for the region.

---

[1] Berwick, V. K., Floods in Eastern Montana Magnitude and Frequency, Open File Report U.S. Geological Survey, Water Resources Division, Helena, MT May 1958 (in cooperation with Montana Highway Commission).

Multiple regression was used to obtain the following prediction equation for the mean annual flood for any watershed in the region using data from 14 of the drainage areas

$$Q_{2.33} = 6.21 \times 10^{6} \quad A^{.533} E^{-1.45} \tag{1}$$

where $Q_{2.33}$ = the mean annual flood in cfs
A = drainage area in sq mi
E = mean basin elevation, ft msl

The standard error of estimate for this equation was reported to vary from +19% to -16% for the 14 watersheds. Presumably this was standard error in logarithms of flow expressed as a percent of the mean logarithm.

Recognition by Montana highway engineers of the lack of sufficient data in Montana upon which to base predictions of flood events from small watersheds led to the establishment on July 1, 1955 of a cooperative program of crest-stage stream gaging by the U. S. Geological Survey, Helena, with important financial support provided by the Montana Highway Commission, Planning Survey Section. Under this program, peak-flow data have been collected for various periods at 214 different sites. Originally 45 crest-stage gaging stations were established. The program was expanded to 138 stations in 1959, and to 202 stations in 1963. About 200 stations were operated from 1963 to 1967, and 185 stations were operated from 1968 through part of 1970. During the fall of 1971 a total of 183 such stations were in operation. This network consisted of 24 stations with drainage areas less than 1.0 sq mi, 92 stations having drainages from 1 to 10 sq mi, and 50 stations having areas from 10 to 50 sq mi. Only 16 crest-stage stations are at sites draining over 50 sq mi.

In addition to the stations operated through cooperation with the Highway Commission the U. S. Geological Survey operates 14 crest-stage stations in eastern Montana in cooperation with the Montana Water Resources Board.

In 1963 Fred C. Boner[2] presented a second U. S. Geologic Survey Interim Report on the Frequency and Magnitude of floods in Eastern Montana, based upon the more extensive flood records then available. The flood records at 40 stations, 21 of which were long term recording stations, and 19 crest-stage stations were used by Boner. As in Berwick's study, composite flood-frequency curves were developed using the records from the 21 long term records for the base period 1938-61. From a comparison made between the long-term stations and the crest-stage stations for the common 6-year period 1956-61, it was concluded that the composite frequency curve developed for the long-term stations was applicable to the other watersheds in the region having the shorter records and smaller drainage areas. The region covered by Boner extended further west than Berwick's to longitude 109°W on the north, and about 109.5°W on the south, but remaining about 110.7°W near White Sulphur Springs. Boner also covered all of the southeastern corner of the State which Berwick had excluded.

Boner made a multiple correlation study to predict mean annual flood as follows:

$$Q_{2.33} = 2.78 \times 10^5 A^{.382} E^{-1.255} L^{.423} G \qquad (2)$$

where $Q_{2.33}$ = mean annual flood, cfs

A = drainage area, sq mi

E = average of elevations at .2 and .8 meander length of main stream channel, ft msl

L = meander length main stream course in miles from Army Map Service topographic map (1:250,000 scale)

---

[2] Boner, Fred C., Frequency and Magnitude of Floods in Eastern Montana, U.S. Geological Survey, Helena, MT (in cooperation with Montana Highway Commission), November 1963.

G = a geographic factor having three values, ranging from 1.00 in the northwest, 2.02 throughout most of the region, to 3.05 along the eastern State boundary.

Boner states that the standard error of estimates for the above equation ranged from +37% to -27% of the mean (again presumably on the logarithms of the flood flows).

Boner and Omang presented a report[3] on floods from drainage areas less than 100 sq mi in Montana in 1967. This report covered all of the State and presented a method for determining the magnitude of floods with 10 and 25-year recurrence intervals. The State was divided into 13 regions, A-M, 4 for the Columbia drainage, one for the Hudson Bay drainage, and 8 covering the Missouri drainage. Peak flow records 5 years or more in length from 160 continuous-record gaging stations and 123 crest-stage and partial record gaging stations were used. A flood frequency curve was developed for each station plotting recurrence interval in years vs flood peak on Gumbel extreme value probability paper. Recurrence intervals were computed by the familiar equation $T_p = (n + 1)/m$ where n = number years of record and m = the relative order of size of flood in the series. From the flood frequency curve the 10-year recurrence interval flood was selected as a so-called index flood for each watershed.

Multiple correlation analyses were made for each region to determine a relation between the 10-year flood and basin characteristics such as: area, mean altitude of basin, channel slope, and average annual runoff. The 13 regions were determined by trial and error groupings of stations until regions were found where the 10-year flood correlated reasonably well with size of drainage area. In the mountainous portion of Montana it was found that the prediction equations developed expressed the 10-year flood in terms of drainage area size and mean annual runoff. Only drainage area was found significant as an independent variable in the remainder of the State.

---

[3] Boner, F. C. and R. J. Omang, Magnitude and Frequency of Floods from Drainage Areas less than 100 Square Miles in Montana, U.S. Geological Survey, Helena, MT Open File Report, 1967.

Median values of the ratio of 25-year floods to 10-year floods were developed for each region using only those records having a long enough record to adequately define the 25-year flood. These ratios varied from 1.2 to 1.5 with the highest value for region F which covered an area about 100 miles wide on the east slope of the continental divide north of approximately 47° latitude.

Boner and Omang regarded the reliability of their definition of the 10-year floods as only fair because of the many records of short length used in their analysis. They rated the reliability of the 25-year flood determination as poor, due to the assumption that the ratios of 10-year floods for stations with a long period of record are essentially the same as for the short-term records. There was of course no way such an assumption could be verified, although if the period of time covered by the short-term records was representative, it appears reasonable.

DRAINAGE CORRELATION STUDY

The final report[4] on a 5-year research study of the hydrology of floods from small drainage areas in Montana was submitted in May, 1971. This two-volume report presented several interrelated studies among which were:

1. A Proposed Rainfall Frequency-Peak Flow Frequency Method
2. Hydrologic Study of Four-Selected Watersheds
3. The 1964 Flood at Lone Man Coulee
4. Multivariate Statistical Studies of Rain Produced Floods
5. Study of the Soil Conservation Service Method of Flood Prediction
6. Review of Other Flood Prediction Methods

---

[4] Williams, Theodore T., Final Report Drainage Correlation Research Project, Vol I and II for the Montana Highway Commission in cooperation with U.S. Department of Transportation, Federal Highway Administration.

Proposed Rainfall Frequency-Peak Flow Frequency Method

The Proposed Rainfall Frequency-Peak Frequency Method was developed by Lee Robinson[5] as a method of estimating flood peaks from ungaged watersheds from 24-hr rainfall records by the formula

$$Q_R^i = \overline{Q}_R\left(\frac{I^i}{\overline{I}}\right)D \tag{3}$$

where Q_R^i = rain produced flood peak of recurrence interval = i years

$\overline{Q}_R$ = mean annual rain produced flood peak

I^i = rainfall intensity for a specified duration for recurrence interval = i years

$\overline{I}$ = mean annual rainfall intensity for same duration (2.33 year Gumbel value)

D = a rainfall-discharge recurrence factor evaluated for 11 watersheds in Eastern Montana for a 50-year recurrence interval

Robinson assumed that the ratio $I^i/\overline{I}$ was constant for a particular rainfall station provided we restrict attention to storms of 12-hr duration or over, so that he was able to construct on a map of Montana isopleths of equal ratio $R = I^i/\overline{I}$ simply by studying daily rainfall records (24-hr durations). He then claimed that the following equation is valid

$$Q_R^i = \overline{Q}_R \frac{I^i}{\overline{I}} \left[\frac{1 + C_{vQ} \dfrac{1}{\sigma_{nQ}} (\ln i - \overline{y}_{nQ})}{1 + C_{vI} \dfrac{1}{\sigma_{nI}} (\ln i - \overline{y}_{nI})} \right] \tag{4}$$

where

C_{vQ}, C_{vI} = coefficient variation of flood flows and 24-hr rainstorms respectively

σ_{nQ}, σ_{nI}, $\overline{y}_{nQ}$, and $\overline{y}_{nI}$ = theoretical values in Gumbels method, which although functions of the lengths of record, are practically constant. σ_n varies from 1.06 for n = 20 years to 1.16 for n = 50 years, while $\bar{y}_n$ varies from .52 for n = 20 years to .56 for n = 100 years. It is clear that D can be evaluated from the expression in square brackets above. From Eq. (1) D must also satisfy the relation

---

[5] Robinson, Lee, Frequency of Peak Flows Predicted from Rainfall Frequencies, Ph.D. Thesis 1968. Montana State University, Department of Civil Engineering and Engineering Mechanics.

$$D = \frac{\bar{I}}{I^i} \frac{Q_R^i}{\bar{Q}_R} \qquad (5)$$

There is no denying the appeal inherent in a method which claims to estimate flood frequencies from rainfall frequencies. It is believed that Robinson's reasoning is faulty. His method assumes that the recurrence intervals associated with both the rain event and the runoff coefficient (infiltration event in Robinson's paper) are single-valued and unique. Recurrence intervals are a measure of probability. Recognizing that I and C in the familiar rational formula

$$Q = C^k I^i A \qquad (6)$$

are both random variables, each following some particular frequency distribution, with its recurrence interval denoted by superscripts k and i, and that these variables are relatively (although probably not completely) independent, we see that a wide range of combinations of C and I may combine to produce a particular value of Q. To illustrate, a particular 15-year flood may be produced by a 10-year rainfall intensity in combination with a 3-year runoff coefficient. However, the same 15-year flood could possibly have resulted from a 5-year rainfall intensity and a 10-year runoff coefficient, or some other combination of an almost limitless set of combinations. For this reason it appears meaningless to try to define a relationship between Q_R^i and I^i as Robinson does in Eq. (4) above.

It is submitted that Robinson's Eq. (3) is really nothing but the rational method which was first presented in the U. S. in 1889[6] by Kuichling and earlier by Mulvaney in Ireland in 1851[7]. The rational method is no longer given much credence for natural watersheds. Consider the equations

---

[6] Kuichling, E., The Relation Between the Rainfall and the Discharge of Sewers in Populous Districts, Transactions ASCE, Vol 20, 1889, p 1.

$$Q_R^i = C_w I^i A \tag{7}$$

$$\overline{Q}_R = C_D \overline{I} A \tag{8}$$

where C = runoff coefficient considered a function only of antecedent moisture conditions. It appears reasonable to assume that $C_w > C_D$, and that the ratio C_w/C_D increases as the recurrence interval becomes greater, (larger floods tend to occur with greater antecedent moisture).

Dividing Eq. (7) by Eq. (8) we have

$$\frac{Q_R^i}{\overline{Q}_R} = \frac{I^i}{\overline{I}} \frac{C_w}{C_D} \tag{9}$$

Comparing Eq. (9) with Eq. (3) we see that the term D in Robinson's Eq. (3) may be regarded as simply the ratio of the two runoff coefficients, C_w/C_D. This ratio is not a unique function of flood or rainfall recurrence intervals. So again we arrive at the conclusion that it is not possible to predict the frequency of flood events on a given watershed solely from rainfall intensity events. This conclusion is not surprising, engineers have generally recognized that the runoff coefficient is variable for a given watershed and cannot be solely related to the magnitude of the rainfall intensity, even for a given location.

Although the Robinson equation appears to have little theoretical basis, a test study was conducted to determine whether actual flood data showed any agreement with the relationship. An area in eastern Montana illustrated in Fig. 1 was chosen as suitable for this purpose. This particular area was chosen because it contains a relatively dense concentration of watersheds

---

[7] Dooge, James C. I., The Rational Method for Estimating Flood Peaks. Engineering, September 6, 1957, p 311. Dooge states that the rational method was proposed and clearly set out by Thomas J. Mulvaney, an Irish engineer in 1851.

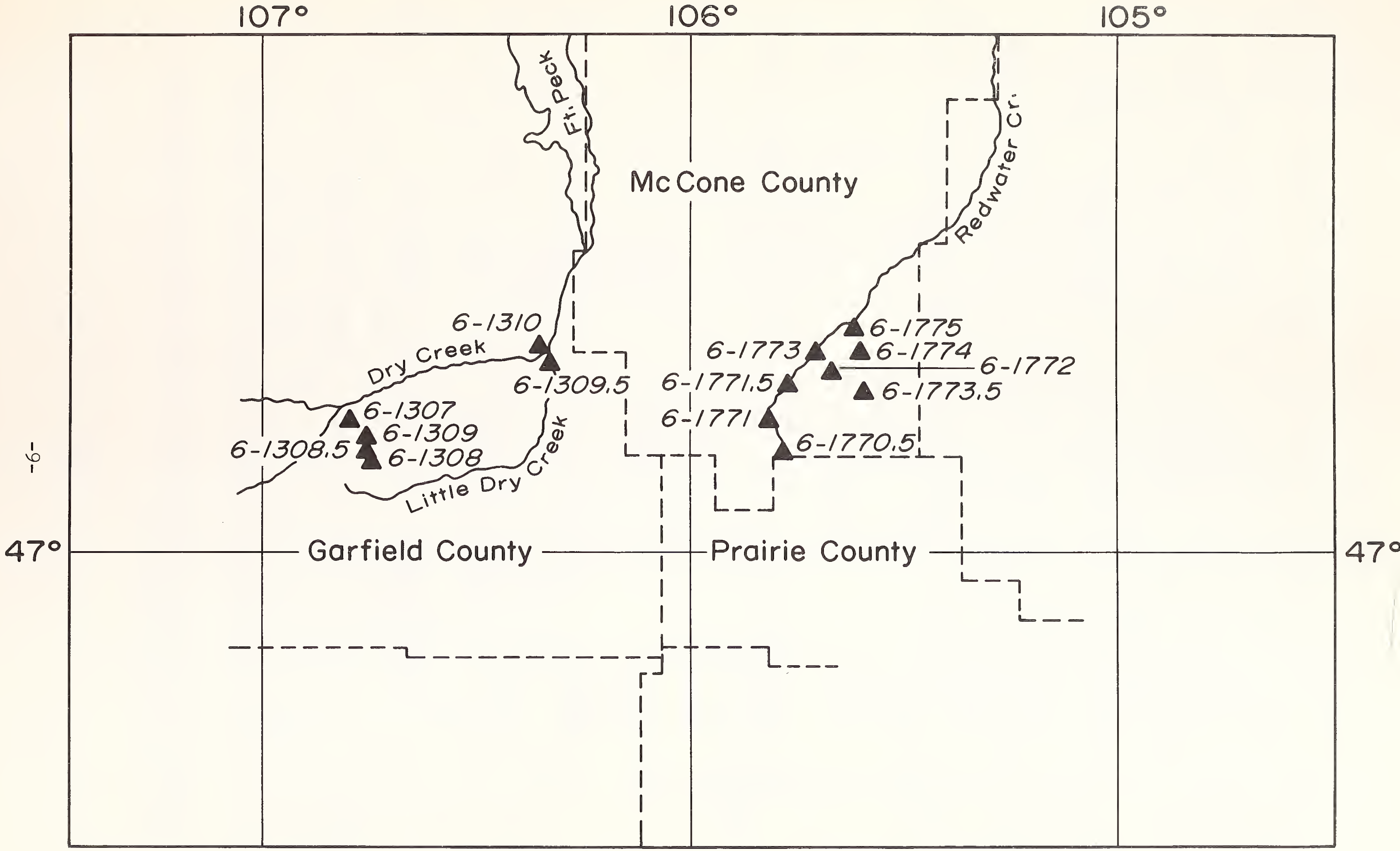


Fig. 1 Map of Area used to Test Robinson Formula

located in a region containing no especially rugged or mountainous terrain and the area appears relatively homogeneous as to general soil type. Hopefully snowmelt effects on flood peaks would be fairly similar in the region, a condition agreeing with Robinson's assumption of uniform snowmelt over all of eastern Montana. As shown 14 watersheds were included in the study area. For each the mean annual flood, the 10-year flood, and the 50-year flood were determined from the log-Pearson computer output based upon a statistical analysis of all available annual flood data for each station. Similarly rainfall intensities for recurrence intervals of 2, 10, and 50-years were determined from the 1969 ESSA maps and U.S. Weather Bureau Technical Paper No. 40. A reasonable time of concentration for each watershed was estimated from Fig. 2, the Kirpich[8] nomograph with the length of drainage area estimated by the formula

$$L = \sqrt{2A} \qquad (10)$$

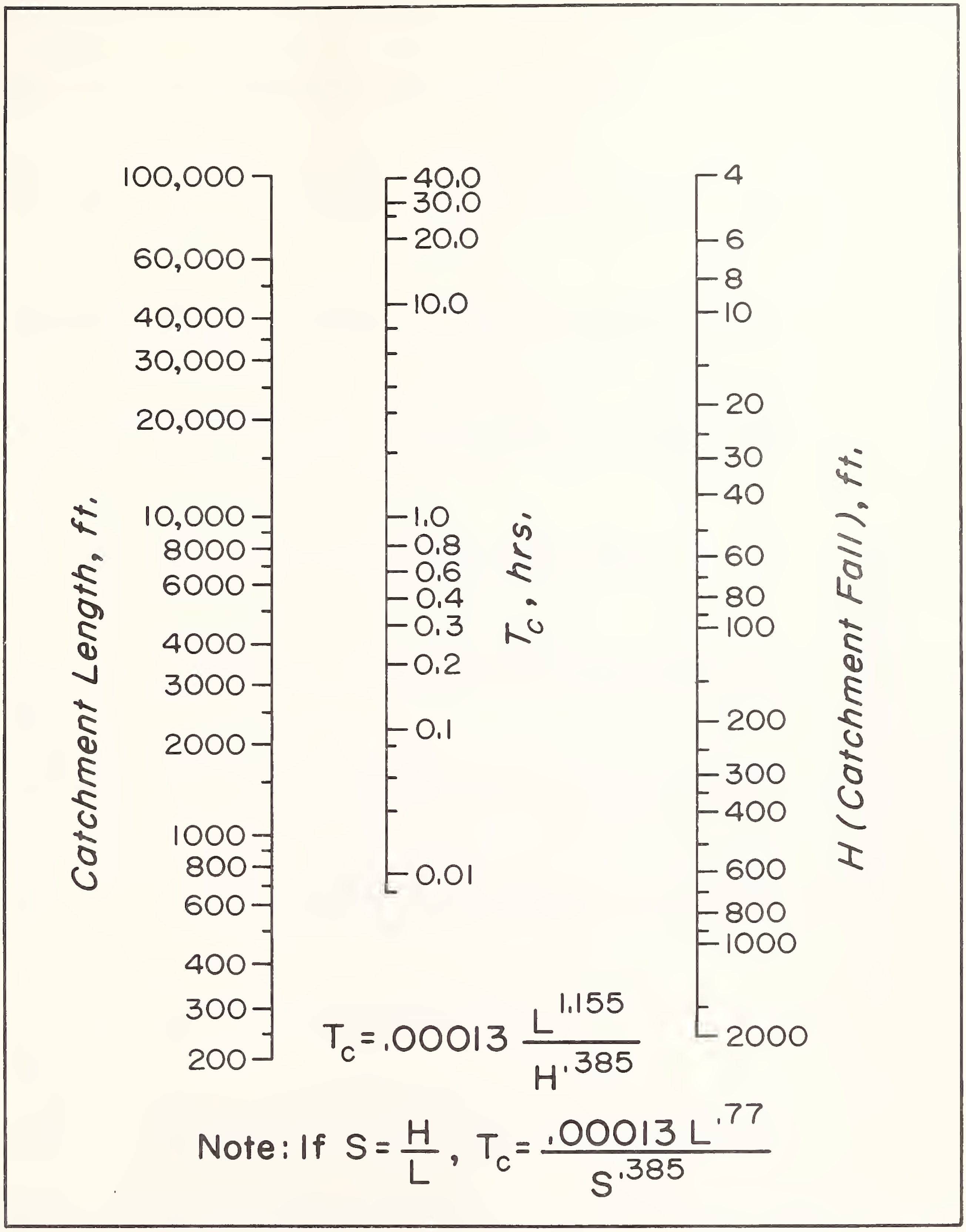
After determination of $\overline{I}$, I^{10}, I^{50}, $\overline{Q}_R$, Q_R^{10}, and Q_R^{50}, values of D were calculated from Eq. (5) which is,

$$D = \frac{\overline{I}}{I^i} \; \frac{Q_R^i}{\overline{Q}_R} \qquad (5)$$

The values of D corresponding to the 10-year and 50-year floods as well as all data from which they were calculated are shown in Table 1. We see that values of D vary from a low of 1.61 to a high of 14.1. In general the values of D derived for 50-year floods are higher, and in four of the watersheds they are from 4-5 times the 10-year D values. D is not at all constant for the region, even for a given recurrence interval. The coefficient of variation, tabulated on the last line, is a statistical measure of the extent of variability. As a matter of interest, values of the flood ratios $Q^{10}/\overline{Q}$ and $Q^{50}/\overline{Q}$ are also tabulated, as are their coefficients of variation. It is seen that the D values vary about as much as these flood ratios, and are therefore about as difficult to predict as the flood ratios themselves,

---

[8] Kirpich, Z. P., Time of Concentration of Small Agricultural Watersheds, Civil Engineering 10:132, 1940.



$$T_c = .00013\,\frac{L^{1.155}}{H^{.385}}$$

Note: If $S = \frac{H}{L}$, $T_c = \frac{.00013\,L^{.77}}{S^{.385}}$

Fig. 2 Kirpich Nomograph for Estimating Time of Concentration, T_c

TABLE 1

Values of D for 10 and 50 year Floods
14-Station Test Area Eastern Montana

Station	Drainage Area mi^2	T_c hr	$\overline{Q}$ cfs	$\overline{I}$ in/hr	I^{10} in/hr	I^{50} in/hr	$Q^{10}/\overline{Q}$	$Q^{50}/\overline{Q}$	D^{10}	D^{50}
6-1307	317.	12	354	.106	.199	.267	6.14	7.02	4.15	3.49
6-1308	.75	.9	26	.66	1.33	1.78	6.53	37.1	3.22	13.70
6-13085	1.90	12	25	.55	1.08	1.46	6.88	35.5	3.49	13.40
6-1309	.60	.8	17	.71	1.44	1.94	5.27	35.7	2.61	13.11
6-13095	1224.	24	1432	.060	.112	.150	3.14	4.96	2.01	2.33
6-1310	2554.	24	2643	.060	.112	.150	5.30	10.75	3.39	5.04
6-17705	12.4	3	108	.32	.60	.80	4.29	7.25	2.33	2.86
6-1771	54.0	5.5	127	.20	.38	.51	8.15	16.14	4.33	6.35
6-17715	216	8.	469	.153	.28	.38	3.40	5.40	1.85	2.16
6-1772	90.2	6.	125	.190	.35	.475	3.08	4.02	1.67	1.61
6-1773	.29	.6	12	1.00	1.95	2.61	7.18	36.9	3.68	14.10
6-17735	5.74	2.	29	.42	.80	1.10	3.14	4.50	1.64	1.72
6-1774	29.9	4.	86	.255	.48	.65	6.27	13.05	3.35	5.11
6-1775	547.	12.	718	.107	.204	.275	7.19	10.06	3.77	3.90
					Means		5.42	16.33	2.96	6.35
					Standard Dev		1.68	13.05	.895	4.76
					Coefficient Var		.31	.80	.30	.75

which are the desired result. From Eq. (3), in altered form

$$\frac{Q_R^i}{\overline{Q}_R} = \frac{I^i}{\overline{I}} D \tag{11}$$

we see that if the ratio $Q_R^i/\overline{Q}_R$ shows no more variability than D, we are gaining nothing through the use of the $I^i/\overline{I}$ ratio. Stated in another way this says that the flood magnitudes calculated from the fitted theoretical distribution show practically no correlation with rainfall intensity magnitudes. Further consideration of the Robinson equation was not believed justified in this report.

Comparison Gumbel, Log-Normal, and Robinson Method

Williams showed that the intensity of 24-hr duration rainstorms in Montana are well represented by the Gumbel extreme value distribution. He then made a test to determine whether there was any significant difference in the reliability of three flood prediction methods, namely the Gumbel distribution, the log-normal (Chow), and the Robinson rainfall peak-flow frequency method for 46 selected watersheds in Eastern Montana. For this test the 50-year recurrence interval flood was predicted for each watershed by each of the three methods. The sum of squares was computed for all watersheds.

A one-way classification Analysis of Variance showed no significant difference at the 99% confidence level between the three flood prediction methods, when the three methods were tested together, and when they were tested in pairs. Since the 46 different watersheds used in this test provide a wide variation in size as well as other watershed parameters, it would appear much better to make a two-way classification analysis of variance so as to eliminate the watershed effect. Such a test, made using the three watersheds together, gives an F value of 1.93 as the ratio of methods mean square to error mean square for (2,90) degrees of freedom. The value of F is significant at about the 15% level, so that no definite conclusion may be made. In other words there is some evidence that the prediction methods are different, although it is not satisfactorily conclusive.

Hydrologic Study of Four-Selected Watersheds

Chapter 3 of the drainage correlation project covered the comprehensive hydrologic study of four widely separated watersheds all east of the Continental Divide in Montana. Table 2 presents a listing of these watersheds with the drainage area, size, and length of record which was available.

TABLE 2

Drainage Correlation Project Watersheds

U.S.G.S. No.	Name	Area, mi^2	Length Record
6-1003	Lone Man Coulee nr Valier	14.6	1960 to-date
6-1208	Antelope Cr Trib No 2 nr Harlowton (Bacon Cr)	21.2	1956 to-date
6-1771	Duck Creek nr Brockway	54.0	1957 to-date
6-2017	Hump Creek nr Reed Point	7.6	1960 to-date

These watersheds were selected on the basis that they had already been included in the U. S. Geological Survey crest-stage streamgaging program, and because they were considered of particular interest and representative of a particular region from a hydrologic standpoint. These watersheds were instrumented quite elaborately. The instrumentation included eight water level recorders to augment the crest-stage device, eight recording type raingages, ten non-recording type raingages, and eight weather stations. The latter consisted of instrumentation which measured and recorded wind speed, wind direction, air temperature, soil temperature, and soil moisture. During the winter months regular snow survey measurements were also taken on the watersheds. Detailed physical characteristics of the four watersheds were determined including drainage area, drainage area shape, elevation of basin, slope of stream channel, stream channel length, stream density, stream orientation, pondage, and land use. Soil analyses were made including medium intensity soil surveys by soil scientists of the Soil Conservation Service and infiltrometer studies made with the mobile rain drop applicator of the Agricultural Research Service, USDA, at 69 different locations of the four watersheds. Approximately six years of climatological and hydrological data were obtained during the duration of the project.

The intensive instrumentation and observations made over the 6-year period on these watersheds provided valuable experience to the project personnel, and did provide some hydrologic data of significance which otherwise would not have been obtained. The most significant of these hydrologic events was probably the 1964 flood which was recorded on the Lone Man Coulee watershed near Valier.

The 1964 Flood at Lone Man Coulee

Chapter 4 of the Drainage Correlation Project covers the 1964 flood at Lone Man Coulee. On June 7-8, 1964 Northwestern Montana had the most severe floods of record on both sides of the Continental Divide as a result of a 36-hr rainfall which produced over 16 inches of water near its center and up to 12 inches along a band just east of the Continental Divide extending from about the latitude of Great Falls to practically the Canadian border. Peak discharges on streams in the flood area range from about 2 to 11.5 times the probable 50-year flood.[9]

This great rainstorm amounted to about five inches on the Lone Man Coulee watershed in about 30 hours with a resulting runoff of 1.67 inches. The estimated peak discharge for this 14.6 square mile area was 1740 cfs or about 120 cfs per square mile. Although this flood discharge represents a rather rare magnitude for this location, unit discharges as high as about 500 cfs per square mile were estimated for some drainage areas as large as 100 sq mi during the 1964 Montana flood.

A unit hydrograph derived from this flood, treating the rainfall as a single storm, had a peak of 1040 cfs and a lag of 11 hours, as measured from the centroid of rainfall to the peak of the hydrograph. The U.S. Geological Survey estimates that the 50-year recurrence interval flood for Lone Man Coulee would have a magnitude of about 700 cfs, or about 40% of the 1964 flood.

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[9] Boner, F. C. and Frank Sternmitz, Floods of June 1964 in Northwestern Montana, Geological Survey Water Supply Paper 1840-B.

The most tragic and spectacular event of the flood was probably the failure of Swift Dam on Birch Creek which released about 30,000 acre feet of stored water in a very short time. The instantaneous peak discharge which resulted 17 miles downstream from the dam was estimated at about 880,000 cfs. The flood wave moved so rapidly that very few people received advance warning. In all 19 residents along the creek lost their lives.

It is interesting to note that during the duration of the Drainage Correlation Project four flood events on Lone Man Coulee were significant enough to permit the derivation of unit hydrographs. However, in each case the rainfall duration was much longer than a value reasonable for computation of a unit hydrograph where the rainstorm is treated as a single event. The lag times for this watershed for these rainstorms of relatively long duration (from 11 to 50 hours) were all about 11 hours.

Multivariate Statistical Studies of Rain Produced Floods

Chapter 5 of the Drainage Correlation Report covers the multivariate statistical studies which were made using the watershed characteristics and the hydrologic data determined from the four project watersheds. By considering East Duck Creek and Duck Creek as separate watersheds this analysis was made on a five watershed basis. The analysis used 12 watershed parameters for each basin, each of which was assumed to remain constant with time. Seventeen storm variables in all were used, making a total of 29 independent watershed and storm variables. In all, 50 runoff events which had been recorded, were used in the analysis.

Principal component analysis and varimax rotation of the principal factors were performed. Regression equations were ultimately developed to express flood peaks in terms of the 29 independent variables. Of the 50 runoff events, 30 were rain caused. Final results for the 30 rain caused events were expressed with a regression equation in terms of 18 selected variables.

While the multivariate statistical study was quite ambitious, it appears that the results have little practical significance because of poor selection of too many independent variables. This appears particularly true of the so-called storm variables. For instance one of these, intensity of precipitation, was computed by averaging the rate of precipitation from the beginning of the storm at the first recording station on the watershed to the hour of last runoff from the hydrograph. It is difficult to see why an intensity computed in this way would be expected to correlate with flood peak. Experience has shown that the use of too many independent variables in regression analysis usually defeats the purpose of the analysis. The author is of the opinion that this unfortunately happened in the multivariate study.

Study of Soil Conservation Service Method of Flood Prediction

Chapter 6 of the Drainage Correlation Report describes an application of the U. S. Soil Conservation Service method in an attempt to predict the flood event of June 13-16, 1965 which occurred on the Duck Creek project watershed near Brockway. The so-called Project Formulation Program-Hydrology computer program developed by the Soil Conservation Service was selected and tested for its ability to predict this flood event. A great amount of detailed field information was obtained for input to this computer program. This included data on stream channel slopes, stream cross sections, and much detail on soil types classified according to the Soil Conservation curve number system.

Average total precipitation during the rainstorm was 3.72 inches and total surface runoff measured 0.35 inches. The storm was considered complex and therefore difficult to model. Two computer runs were made from which peak flow predictions were obtained. In Run No. 1 the calculated surface runoff was .57 inches with a resulting peak flow of 1144 cfs. For Run No. 2 the surface runoff amounted to .43 inches and the peak flow 1006 cfs. It was apparently not possible to make additional computer runs so the results were manually adjusted to produce a surface runoff which agreed with the measured value of .35 inches and the estimated flood peak for this adjusted computation was 820 cfs, still more than twice as high as the recorded flow of 342 cfs.

It is pointed out in the report that the Soil Conservation Service emphasizes that their method was never intended to reproduce actual flood events, but rather to produce statistical estimates of peak flow for spillway design purposes. It is difficult, therefore, to draw any conclusions regarding the result of this particular study, except that as expected the agreement between the calculated peak and the measured peak is quite poor. It appears that one would have to apply any such prediction method to a considerable number of storms and analyze the results on a statistical basis in order to assess the reliability of the prediction method.

Review of Other Flood Prediction Methods

Chapter 7 of the Drainage Correlation Report describes four miscellaneous flood peak discharge prediction methods. These are the SCS COOK method, the U. S. Bureau of Public Roads (1951) method, the U.S. Bureau of Public Roads Potter method, and the Northern Pacific Railway method. The latter is a highly empirical old-fashioned method wherein the square foot of culvert opening was estimated by multiplying a coefficient times the square root of the drainage area. These coefficients varied from .5 to as high as 6. for the various regions through which the Northern Pacific Railway travelled.

The USBPR Potter method is apparently rather highly regarded by some Federal Highway Transportation personnel. This method as published, however, covered only that portion of the United States east of about longitude 106° and so was not tested for Montana areas. It is our understanding that a research project at Utah State University is presently determining whether this method has validity as applied to other parts of the United States.

A comparison of the several methods of flood prediction available at that time was made in the Drainage Correlation Report. This comparison was made for the five so-called project watersheds using Boner's 1963 USGS method, the Boner-Omang method, the Gumbel method, and the Potter method to predict the 10-year and 25-year recurrence interval floods. Along with the predicted floods for each watershed were tabulated the mean annual flood as calculated from the Gumbel distribution (2.33 yr recurrence interval) and the peak flood of record for the watershed. It should be noted that the length of record for these project watersheds varied from 10 to 12 years.

This represents a rather short time span on which to base conclusions regarding the reliability of flood prediction methods by comparing values predicted for 10 and 25-year recurrence intervals with the maximum values which have been observed. Nevertheless, William's conclusion was that the Gumbel method appeared to be the most reliable for predicting peak flows on the five project watersheds.

Chapter 10 of the Drainage Correlation Report presents a study of four Montana watersheds with the rather long records, varying from 31 to 44 years, for the purpose of comparing flood peak predictions for various recurrence intervals by the three most commonly used frequency distributions. These were (1) the Gumbel extreme-value method, (2) the log-normal (Chow) method, and (3) the log-Pearson Type III method. For each watershed and method, peak discharge magnitude and frequency was predicted using only the first half of the record and then using only the second half of the record. The same flood frequency values were then predicted using the entire record. Apparently the criteria being used for selection of the more reliable flood prediction method was that the most reliable method would show the least difference when predicting peak flows with a 50-year recurrence interval of any one of the three methods.

Williams concluded from this study that the log-Pearson Type III method was most satisfactory. With this method the divergence between the curves at the 50-year recurrence interval was considered satisfactory on three of the four watersheds. For the Gumbel method the divergence was considered satisfactory on only two of the watersheds, while for the log-normal method the divergence was considered satisfactory on only one watershed. It should be noted that this method assumes a stationary time series, since changes in climatological or hydrological conditions from the first half of the record to the second half of the record would obviously cause discrepancies in the predictions which would not be indicative of non-reliability of the prediction method. Williams recognized that considerable discrepancy can be expected when predicting peak flows at the 50-year recurrence intervals from records as short as 10 or 15 years..

Chapter II

HYDROLOGIC STUDIES

SCOPE OF DATA

The streamflow records in Montana considered suitable for a statistical study of floods consisted of those stations with a minimum of approximately 10 years of record for which upstream diversions did not have an important effect on flood peaks. Such stations had been selected by the Helena office of the U. S. Geological Survey as a basis for the report "A Proposed Streamflow-Data Program for Montana" by Boner and Buswell (1970). From that report a total of 106 continuous-record, 18 seasonal-record, and 106 crest-stage (peak-flow), stations were selected as giving suitable data on annual flood peaks for these 230 watersheds.

Fig. 3 shows on a map of Montana outlines of all drainage areas used in this study.

Based on Montana's area of 147,138 sq miles this represents an average coverage of 640 sq mi per watershed. However the actual average area of the 230 watersheds is about 370 sq mi. Actually the extent of the flood-gaged coverage of the State is considerably poorer that this indicates for two reasons, (1) the watersheds selected drain a considerable area of Canada just north of Montana's border as well as some of Wyoming on the south, and (2) in several instances watersheds lie within watersheds giving rise to more intensive coverage of the largest watershed, but poorer average coverage of the State. For example, station 6-1775, Redwater River at Circle gages 547 sq miles, but part of this same area is gaged by 7 other stations. Thus the average coverage within 6-1775 could be said to be 547/8 or about 70 sq miles per station but with regard to the entire State none of these 8 stations provide coverage of areas outside watershed 6-1775.

Tables 3 and 4 show a tabulation of all watersheds selected classified by major drainage areas, according to type of record, and according to size of drainage area.

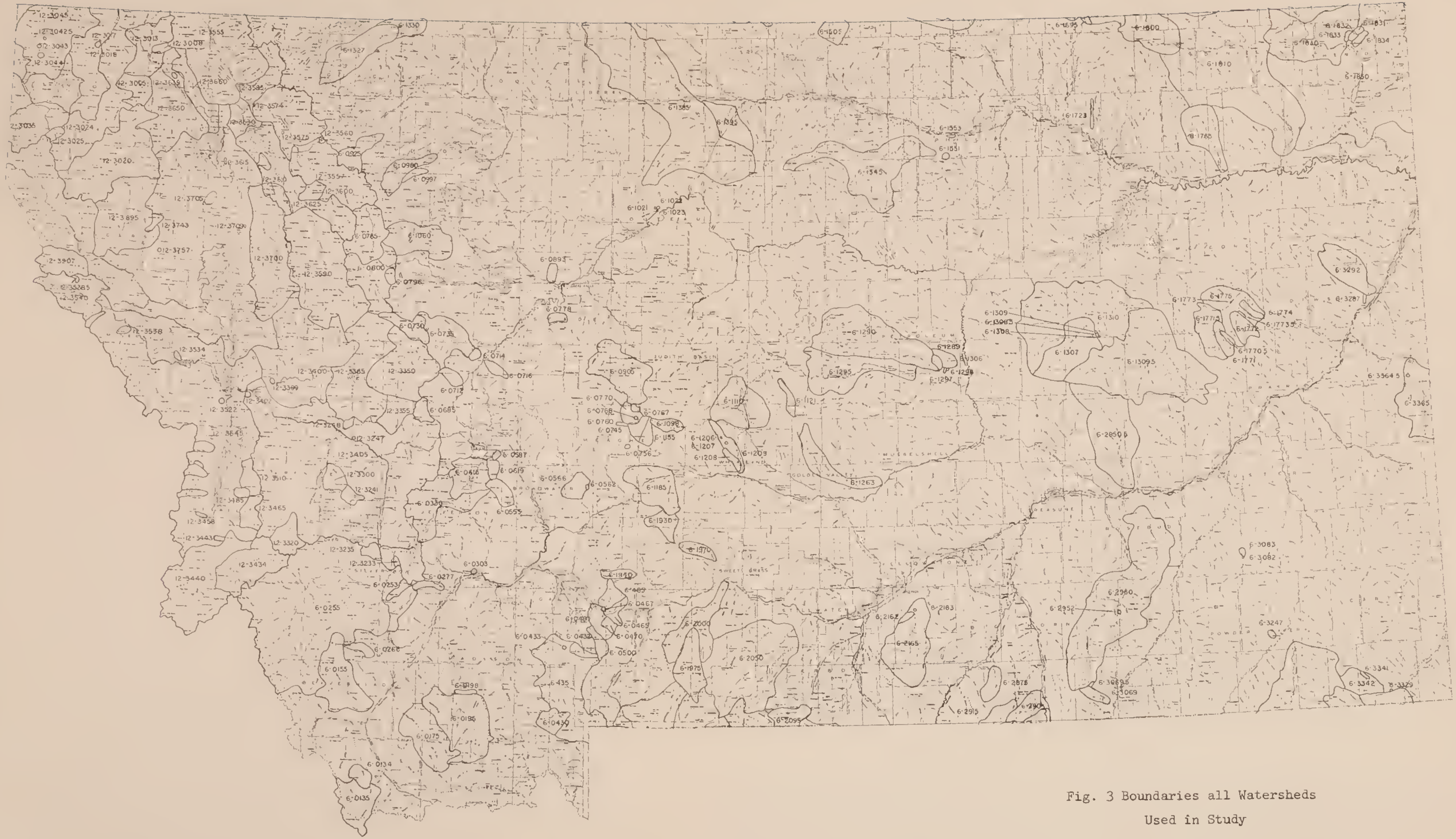

Fig. 3 Boundaries all Watersheds Used in Study

All annual floods as selected by the Helena U.S. Geological Survey office for each watershed for the period 1920 through 1969 were used except the 1964 flood value for certain watersheds as discussed under WATERSHED FLOOD-FREQUENCY DATA. From Tables 3 and 4 we see that almost half of the records (106 out of 230) were obtained at peak-flow stations, the so-called crest-stage sites established by the U.S. Geological Survey in cooperation with the Montana Highway Department, beginning in 1955. In 1970 a total of 183 of these gages were in operation in Montana, but only 102 of these had records of about 10 years or more since four of the 106 tabulated in Table 3 had records less than 8 years.

Table 5 presents length of record data for all the 230 watersheds used. It is seen that 129 out of the 230 or 56% have records of 15 years or less, whereas 52 stations or 22% have records over 25 years. The longest records are 48 years, for the Madison River near West Yellowstone, and for the Yellowstone River near Corwin Springs.

Three large Columbia Basin watersheds included in the above tabulations with drainage areas of 5,999; 10,240; and 10,709 sq mi respectively were not used in the Regression Analysis, since their size was so much larger than the scope of the objective of this report.

Since an objective of this study was to provide estimates of floods with recurrence intervals up to 50 years for any location in Montana, it was concluded that the use of some sort of regional analysis wherein short records can be extended and records with missing years can be filled in with estimated flood flows would be desireable. The estimated flood flows could hopefully be calculated using correlations developed from recorded chronologic flows at other stations in a given region. We obtained the Regional Frequency Computation Computer Program 23-X6-L268 developed at the U. S. Army Corps of Engineers Hydrologic Engineering Center at Davis, California through the courtesy of Mr. Leo Beard, Director of the Center.

TABLE 3

Classification Watersheds by Type Records

Drainage	Type Record			
	Continuous	Seasonal	Peak-Flow	Total
Missouri River	60	18	76	154
Columbia River	39	0	30	69
Hudsons Bay	7	0	0	7
Totals	106	18	106	230

TABLE 4

Classification Watersheds by Size

Drainage	Watershed Size, sq mile						
	0-5	5^{+}-15	15^{+}-50	50^{+}-100	100^{+}-200	200^{+}-500	>500
Missouri River	23	16	26	21	12	31	25
Columbia River	11	13	12	4	5	6	18
Hudsons Bay	1	0	2	2	1	1	0
Totals	35	29	40	27	18	38	43

TABLE 5

Summary of Annual Flood Peak Records Lengths in Years

	Record Length in years (inclusive)					
Drainage	<8	8-15	16-25	26-35	over 35	Total
Missouri	4	83	31	17	19	154
Columbia	0	40	15	7	7	69
Hudsons Bay	0	1	3	0	3	7
	4	124	49	24	29	230

REGIONAL ANALYSIS

PROGRAM DESCRIPTION

The Corps of Engineer's regional frequency program[10] was designed to develop frequency (recurrence interval) relations for annual maximum hydrologic events, such as stream flows for various time durations for each of a group of stations in a hydrologic region. When two or more of the stations have long records, it is considered to provide a desireable method for optimizing flood frequency predictions for a region, particularly if many of the other records are of relatively short length.

The program uses the Pearson Type III frequency distribution which deals with logarithms of the flow events. It permits the use of zero-value flood events by virtue of the addition of .1 percent of the average flow to each value entered before logarithms are calculated, thus avoiding the indeterminancy of negative infinity for the logarithm. This increment is later subtracted from estimated flows and the computed frequency values.

The frequency statistics of mean, standard deviation, and skew coefficient are computed for recorded events at each station. Missing events are then estimated preserving all intercorrelations but introducing a random component representing the uncorrelated portion of the estimate. Statistics for each station are adjusted to the complete period of regional record and frequency curves computed, either with the individual calculated station skew coefficients (including estimated events), or if desired an inputed overriding regional skew value. The basic relation for the flood flow of any desired frequency using the Pearson Type III method[11] is

$$\log Q_T = M + K_T S \qquad (12)$$

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[10] Computer Program 23-X6-L268, Hydrologic Engineering Center, U.S. Corps of Engineers, 609 2D Street, Davis, California 95616.

[11] Water Resources Council, "A Uniform Technique for Determining Flood Flow Frequencies" December, 1967.

where Q_T = flood magnitude in cfs

M = mean logarithm of flood events

K_T = number of standard deviations from mean corresponding to recurrence interval (or percent chance of flood) and skew coefficient (K_T values are available in published tables)

S = standard deviation of logarithms of floods

The Fortran program as received accepted annual flood input data with values corresponding to each years data on a separate card. Input formats and logic were modified to permit the annual floods to be punched in the order of their occurrence as successive pairs of values giving year and flow, with 8 pairs per card. If a flood was missing for any year that pair was simply omitted from the card. Other modifications included (1) providing for processing up to 40 stations with a regional maximum of 50 years of record instead of 10 stations, 50 years, and 8 flow durations, (2) rewriting the random number generating routine to fit the Sigma 7 Computer and so it would give a different sequence of random numbers for each run of identical data, (3) modifying some output statements, and (4) adding a linear regression computation to find the least squares best fit of the 50% probability flood (2-year recurrence interval) as a power function of drainage area.

The program estimates missing flows in the following manner:

1. Simple correlation coefficients are calculated between all pairs of stations having at least 3 common years of recorded flows.

2. These coefficients are built into a correlation matrix, which since the stations have different lengths of record, will contain missing and inconsistent values. The missing coefficients are estimated from related pairs of values and low values are raised to obtain consistency. A multiple correlation coefficient is then calculated for each station, with adjustments made to the correlation matrix so that the calculated multiple correlation coefficient is less than 1.00.

3. After converting all flows (actually logarithms of flows) to equivalent normal deviates, the missing flows at each station are estimated from the equation

$$k = \Sigma B_i k_i + \sqrt{1-R^2}\, Z \tag{13}$$

where

k = standard deviate of missing flow

k_i = standard deviate of recorded or estimated flow at station i (may be zero)

B_i = linear regression equation coefficient for station i developed by least squares best fit. The summation is taken over all other stations.

R = multiple correlation coefficient discussed in 2 above. Note if R = 1.00 data indicates missing flow can be reliably predicted completely from flows at other stations, while if R = 0.0 missing flow is predicted entirely from random component.

Z = random number normally distributed

The term $\sqrt{1-R^2}\, Z$ in the above equation is a so-called random component. It introduces as a part of the estimated missing flood, a random flow, the importance of which depends on the extent to which flows at the station in question correlate with recorded or estimated flows at the other stations. With perfect correlation the random component is zero, with no correlation the missing flow consists solely of the random component. The estimated standard deviate is of course converted back to actual flow in cfs before output occurs.

When all flows have been estimated, the frequency statistics for each station are recomputed. Regression lines of standard deviation and skew coefficient versus mean are computed separately, and smoothed values of standard deviation and skew calculated. Equivalent length of record, N, for the recorded and estimated flows is estimated for each station from that at the long record base station by the equation

$$N = \frac{N_1}{1 - \frac{(N_2 - N_1)\overline{R}^2}{N_2}} \tag{14}$$

where N = equivalent years at station 1, N_1 and N_2 = actual years at station 1 and at the base station respectively, and $\overline{R}$ is the multiple correlation coefficient for station 1.

This equivalent record is used in computing expected probabilities. Frequency-curve coordinates for each station are computed from the mean, standard deviation, tabulated values of the normal distribution, skew coefficient, and equivalent record length using the Pearson Type III function and the following equations for expected probability.

$$\begin{aligned}
P_{1000} &= .1(1+280/N^{1.55}) \\
P_{100} &= 1 + 26/N^{1.16} \\
P_{50} &= 2(1+14/N^{1.10}) \\
P_{25} &= 4(1+7.5/N^{1.05}) \\
P_{10} &= 10(1+3.00/N^{1.00}) \\
P_{5} &= 20(1+1.00/N^{.950})
\end{aligned} \tag{15}$$

where P_i = expected probability in percent of i-yr recurrence interval flood

N = equivalent years of record

The P_2 or 2-yr recurrence interval flood is always assumed to have a probability of 50%.

WATERSHED FLOOD-FREQUENCY DATA

Flood frequency data were defined for each watershed, using the Regional Analysis Program. Trial regional groupings of geographically adjacent watersheds were made, based upon regional flood characteristics as shown by previous studies. Each region was limited to not more than 40 watersheds, the capacity of the MSU Sigma 7 Computer Facility using "big core" during the Winter of 1971. An attempt was made to include two or more watersheds with reasonably long records in each region to provide a basis for estimating the missing year floods at all other stations.

If for a year with a missing flood for any watershed there were not at least three other watersheds with recorded floods and non-zero correlation coefficients, the program output statements denoted this fact. Such a missing flow was then estimated as a purely random value. Since we did not wish to generate estimates of missing flows on a random basis, when this occurred every attempt was made to re-group the watershed with other adjacent regional groupings where non-zero correlations could be found.

In general it was found that somewhat fewer than 40 watersheds in a regional grouping existed when a reasonably optimal correlation between watershed records occurred. It was also found that the time required for program execution increased quite rapidly with over 30 watersheds in a region. For these reasons, plus the fact that by reducing the maximum number of watersheds in a region to 32 we could run at any time of day without waiting for "big core," the program was modified to handle a maximum of 32 watersheds in a single geographic region. This provided ample capacity for any regional groupings found to produce flood estimates for missing years which consistently appeared reasonable.

Rather early in the regional analyses it became apparent that the magnitude of the great June 1964 flood peaks for certain watersheds which straddled the continental divide in northern Montana were so great that no reasonable estimate of their recurrence interval was possible. The Geological Survey estimates that peak discharges on streams in the flood region ranged from 2 to 11.5 times the probable 50-year flood.[9] The deletion of these values was decided upon only after careful consideration of the objectives of this report. Such deletion was admittedly a debateable procedure and certainly not one to be advocated where the resulting flood predictions are to be used to design structures where failure might involve human safety. However, since the results of this study are intended solely for use in determining the required capacity of highway culverts, it was judged that the use of the extremely high 1964 floods would unduly bias the results, causing unjustifiable expenditures for many routine highway culvert locations when it appears that design should be based upon our best estimate of recurrence interval, not what might be termed the maximum probable flood.

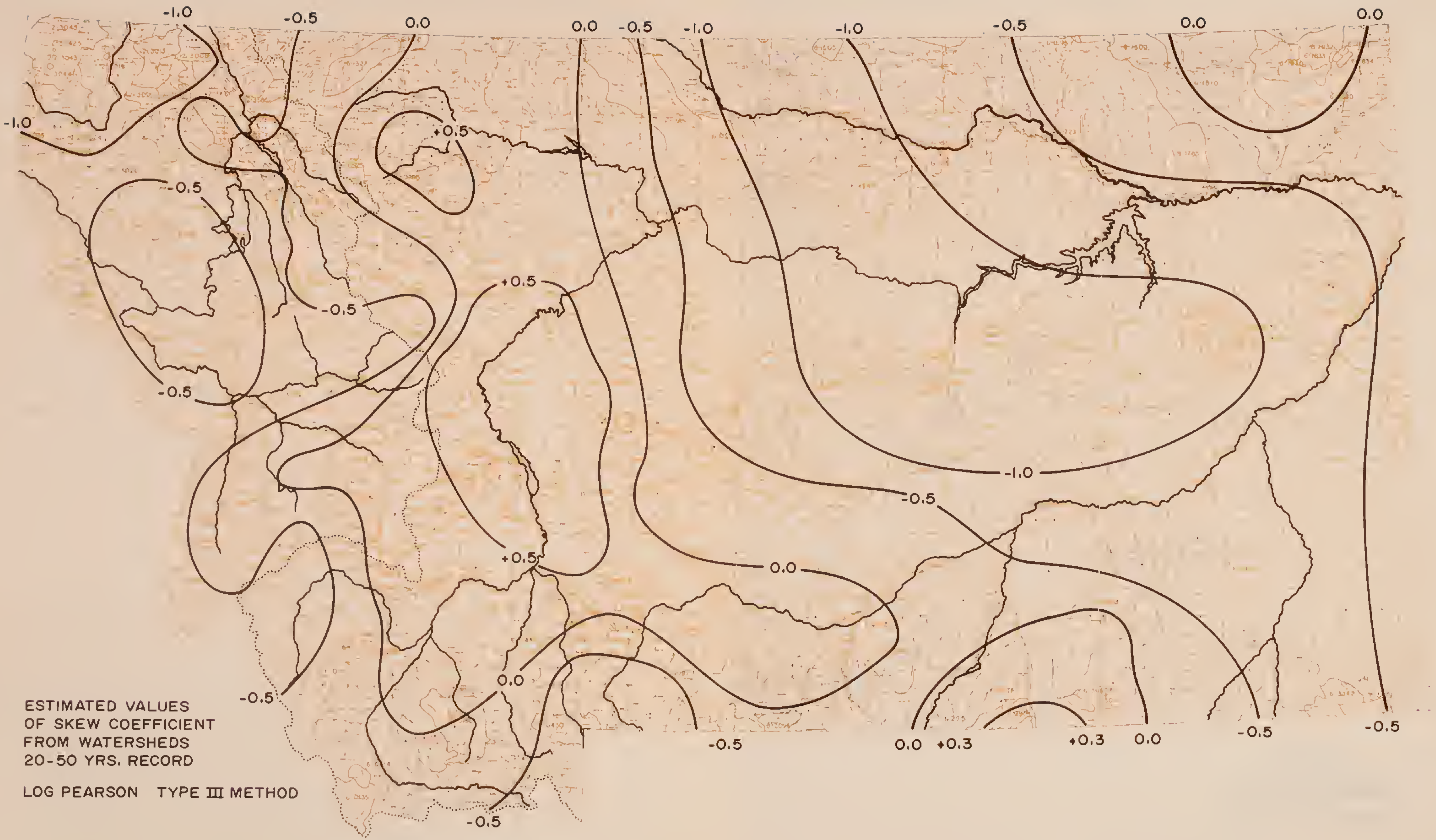
Table 6 lists those stations for which the 1964 flood peaks were deleted. In one or two instances the values may have been used on a regional analysis computer run which was used to estimate flood frequency output for an adjoining station, but they were never used for estimates for the station listed. It appears that there was little justification for deleting the 1964 flood peak at stations 6-735, 6-1327, or 12-3590 since these values only ranged from 1.41 to 1.46 times the estimated 50-year recurrence floods. However, their deletion did not seriously affect the results. The other values ranged from 2.08 to 12.3 times the stimated 50-year flood.

The program permitted computation and use of the skew coefficient resulting from all recorded and estimated floods for each watershed, or as an alternative, the input and use of a desired regional skew coefficient in computing the flood frequency data for each basin. It was found that the skew coefficients based on recorded flows for those watersheds having relatively short records were quite erratic within a given regional grouping and in geographical distribution over the State. Using only those watersheds with 20 years of record or more (58 had records 25 years or longer) skew coefficients were plotted on a map of Montana and iso-skew lines sketched to determine if the data seemed to show any consistency in geographic variation over the State. The map, see Fig. 4, after averaging and shading the data as described below, shows a rather orderly geographic variation, with negative

TABLE 6

List of Watersheds for which 1964 Flood Peaks were deleted

Station	Watershed Description	Area sq mi	1964 Flood cfs
5-100	Belly River at International Boundary	74.8	12,000
5-110	Belly River nr Mountain View, Alberta	121.	16,400
5-115	Waterton River nr International Boundary	61.	12,400
5-125	Boundary Creek at International Boundary	21.	5,930
5-130	Waterton River nr Waterton Park, Alberta	238.	25,700
5-145	Swiftcurrent Creek at Many Glacier	31.4	6,700
6-735	Dearborn River nr Craig	325.	15,400
6-785	North Fk Sun River nr Augusta	258.	51,100
6-796	Beaver Creek at Gibson Dam	20.3	4,360
6-925	Badger Creek nr Browning	133.	49.700
6-997	Middle Fk Dry Fk Marias River nr Dupuyer	20.2	4,240
6-1327	Milk River nr Del Bonita	325.	17,300
12-3555	Flathead River nr Columbia Falls	1548.	69,100
12-3560	Skyland Creek nr Essex	8.1	3,580
12-3585	Middle Fk Flathead River nr W. Glacier	1128.	140,000
12-3590	So. Fk Flathead River at Spotted Bear	958.	36,700



ESTIMATED VALUES
OF SKEW COEFFICIENT
FROM WATERSHEDS
20-50 YRS. RECORD

LOG PEARSON TYPE III METHOD

Fig. 4 Iso-Skew Lines for Montana

values ranging from 0.0 to -1.0 in the eastern prairie area and over the Columbia River drainage and positive values ranging from 0.0 to +0.5 along the Eastern slope of the Continental Divide and 0.0 to +0.3 near the Wyoming border in the mountains southeast of Billings. In contrast to the latter, the mountainous regions of Western Montana seemed to rather consistently show negative skew values, from -0.5 in Yellowstone Park, to -1.0 in the extreme northwest corner of the State. Based upon Beard's recommendation[12] that skew coefficients of zero be used for annual flood peaks except for very long records, the iso-skew lines on Fig. 4 were sketched on the "zero-side" of actual values which varied from -1.8 to +0.8, (omitting the extreme values of +3.5 for a 22 year record and -2.1 for a 20 year record). It is believed that the values shown on Fig. 4 represent more reasonable estimates of skew coefficients for Montana for the log Pearson Type III frequency distribution for peak annual floods than a value of zero for the entire State.

A total of about 100 Regional Analysis computer runs were made using different watershed groups, 25 in the Columbia drainage and about 75 for the Missouri drainage. The 7 Hudson Bay drainage watersheds were grouped in combination with either Columbia and Missouri River basins to determine where they best fit. All acceptable runs for each watershed were tabulated and where the skew coefficient used did not agree with the final adopted values as shown on Fig. 4, the final flood-frequency values obtained were revised to conform to an assigned regional skew coefficient. Finally all flood magnitudes produced for each watershed for the 2-year, 5-year, 10-year, 25-year, and 50-year recurrent intervals using the assigned skew coefficients were tabulated and averaged. These average values were taken as our best estimate of the floods which had occurred for each recurrence interval, with the exceptions shown in Table 7.

These exceptions were watersheds for which some zero flow annual flood events were belated discovered, and/or watersheds for which the average standard deviation appeared abnormally high (except 6-3287) where it was low). The flood-frequency curves based on the values given in Table 7 under Remarks

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[12] Beard, Leo R., Statistical Methods in Hydrology, published by U.S. Army Engineer District, Sacramento, CA 1962, p 6-02.

Table 7

Watersheds for which Regional Frequency output not averaged

Sta No	Watershed	Area, sq mi	Years Recorded	Remarks
6-1263	Currant Cr nr Roundup	220	11	used mean log 2.048, $\sigma_{\log}$ = .6, skew = -.5
6-1385	Big Sandy nr Box Elder	1629	10	record not used, sta 6-1395 appeared more reasonable
6-1795	West Fork Poplar River	139	20	used average mean log, $\sigma_{\log}$ = .5, skew = -.5
6-1700	McEachern Cr at Bdry	182	43	used average mean log, $\sigma_{\log}$ = .5, skew = -.5
6-1832	Box Elder Cr at Dam near Plentywood	199	10	used mean log 2.463, $\sigma_{\log}$ = .5, skew = -.5
6-1833	Spring Cr nr Plentywood	7.1	15	used mean log 1.310, $\sigma_{\log}$ = .5, skew = -.5
6-1834	Spring Cr at Highway 16 near Plentywood	16.9	15	used mean log 1.804, $\sigma_{\log}$ = .5, skew = -.5
6-3287	Linden Cr at Intake	50	10	used mean log .927, $\sigma_{\log}$ = .6, skew = -.5

appeared more reasonable than the values obtained from the Regional Analysis program. Although these adjustments were made somewhat reluctantly, it is believed that they provide more reasonable results. The effect of these engineering judgment adjustments does not alter the regression analysis results very much.

Appendix A tabulates the flood magnitudes mean logarithms (base 10), and standard deviation of the logarithms for each watershed for the five recurrence intervals from 2 years to 50 years, as well as all other watershed parameters used in the Regression Analysis to determine flood prediction equations. In Appendix A watersheds are grouped according to the final nine Flood Regions adopted for the State. Since skew coefficients were assigned on a regional rather than individual watershed basis, they have not been tabulated for each drainage area.

REGRESSION ANALYSIS

FLOOD PREDICTION EQUATIONS

The method decided upon for flood prediction was the development of prediction equations which express the peak flood for a given recurrence interval as a mathematical function of various watershed parameters. Although there are many ways of approaching this problem, the method which proved most successful for this study was the use of the digital computer to perform the statistical technique known as stepwise linear regression. Stepwise regression was chosen primarily because of the versatility it offers in the selection of watershed parameters to be the independent variables in the prediction equations. For example, stepwise regression allows every variable in the regression equation (as well as every variable not in the equation), to be tested for statistical significance each time a new variable is considered for insertion into the equation. In this way variables can be deleted which do not maintain a specified level of significance after they are brought into the equation and new variables satisfying this same level of significance may be added, one at a time. The order in which new variables will be considered for addition may, if desired, be specified. Thus stepwise regression produces final prediction equations in which all variables meet or exceed the desired level of statistical significance.

The various watershed parameters which were used in the regression analysis are listed in Table 8 with the source from which the actual data was obtained. Of the 16 parameters, 10 are measurable physical characteristics of the watershed while the remaining 6 parameters are calculated from mathematical combinations of the physical parameters.

The flood prediction equations for this study were developed through the use of the UCLA (University of California) computer program No. BMD02R.[13] This is a powerful and versatile linear regression program. Aside from the efficient manner in which data may be entered into the program, there are

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[13] Dixon, W. J., Editor, BMD Biomedical Computer Programs, University of California Publication in Automatic Computations No. 2, University of California Press, Berkeley and Los Angeles, 1967.

TABLE 8

List of Watershed Parameters for Dependent Variables in Regression Analysis

No.	Variable Description	Designation	Units	Source of Data
1	Drainage area	A	sq mi	U.S. Geol Survey[14]
2	Average channel slope	S	ft/ft	U.S. Geol Survey
3	Channel length	L	mi	U.S. Geol Survey
4	% Lakes	LK	%	U.S. Geol Survey
5	Mean basis elevation	E	ft x 1000	U.S. Geol Survey
6	% Forest cover	F	%	U.S. Geol Survey
7	Weather Bureau mean annual precipitation	WP	in/year	U.S. Geol Survey
8	% of area greater than 6000 ft	% > 6000	%	U.S. Geol Survey
9	Soil Conservation Service mean annual precip	PS	in/year	SCS Bozeman[15]
10	2 yr-24 hour precipitation	I	in	Weather Bureau map[16]
11	Soil index	SI	in	U.S. Geol Survey and SCS[1]
12	Length/$\sqrt{\text{slope}}$	$L/\sqrt{S}$	mi	Calculated from above
13	12/Area$^{.67}$	$L/\sqrt{S}A^{.67}$	mi$^{-.34}$	Calculated from above
14	% > 6000/Soil Index	% > 6000/SI	in^{-1}	Calculated from above
15	Area/Length^2	A/L^2	dimensionless	Calculated from above
16	Mean annual precip SCS/soil index^2	PS/SI^2	in^{-1}	Calculated from above

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[14] Boner, F. C. and G. W. Buswell, A Proposed Streamflow-Data Program for Montana, open file report, Helena, 1970.

[15] Mean Annual Montana Precipitation in Montana, with Valley Overlays, U.S. Soil Conservation Service, Bozeman, Montana, 1970.

[16] Precipitation-Frequency Maps for Montana, Special Studies Branch Weather Bureau - Environmental Science Services Administration, U.S. Department Commerce, March 1969.

[17] Same as [14] plus additional unpublished values by Mr. R. L. Moshier and Mr. R. Dean, U.S. Soil Conservation Service, Bozeman, MT

other advantages of the program which are valuable to this type of study. Such advantages are that this program provides practically unlimited variable transformation capability and allows any variable to be forced to appear, or blocked from appearing in the equation regardless of the statistical significance of the variable. Because of this it is possible to explore any combination of independent variables even though the particular combination may not have the highest possible statistical significance. For example, after considerable experience in this study it was deemed appropriate to force the drainage area to appear in each regression equation even though, in one flood region, a higher level of statistical significance could have been achieved if the drainage area was omitted and another variable highly correlated to area used instead.

Prediction equations were determined for the 2, 5, 10, 25 and 50 year recurrence interval floods for separate geographic regions of the State. These floods are designated as follows:

Designation	Recurrence Interval T_p years	Probability Occurrence each year, $P = 1/T_p$
Q_2	2	.50
Q_5	5	.20
Q_{10}	10	.10
Q_{25}	25	.04
Q_{50}	50	.02

This involved the repeated use of the regression program in a trial and error process to determine the optimum combination of contiguous watersheds to form a given region and the best location for the boundaries of the region. Nine regions were finally adapted, 2 west of the divide, and 7 east of the divide. The location of the final boundary lines appear in Fig. 5.

LOG-LINEAR vs SIMPLE LINEAR REGRESSION

All prediction equations were developed in the form

$$Q = b_o X_1^{b_1} X_2^{b_2} \dots X_n^{b_n} \qquad (16)$$

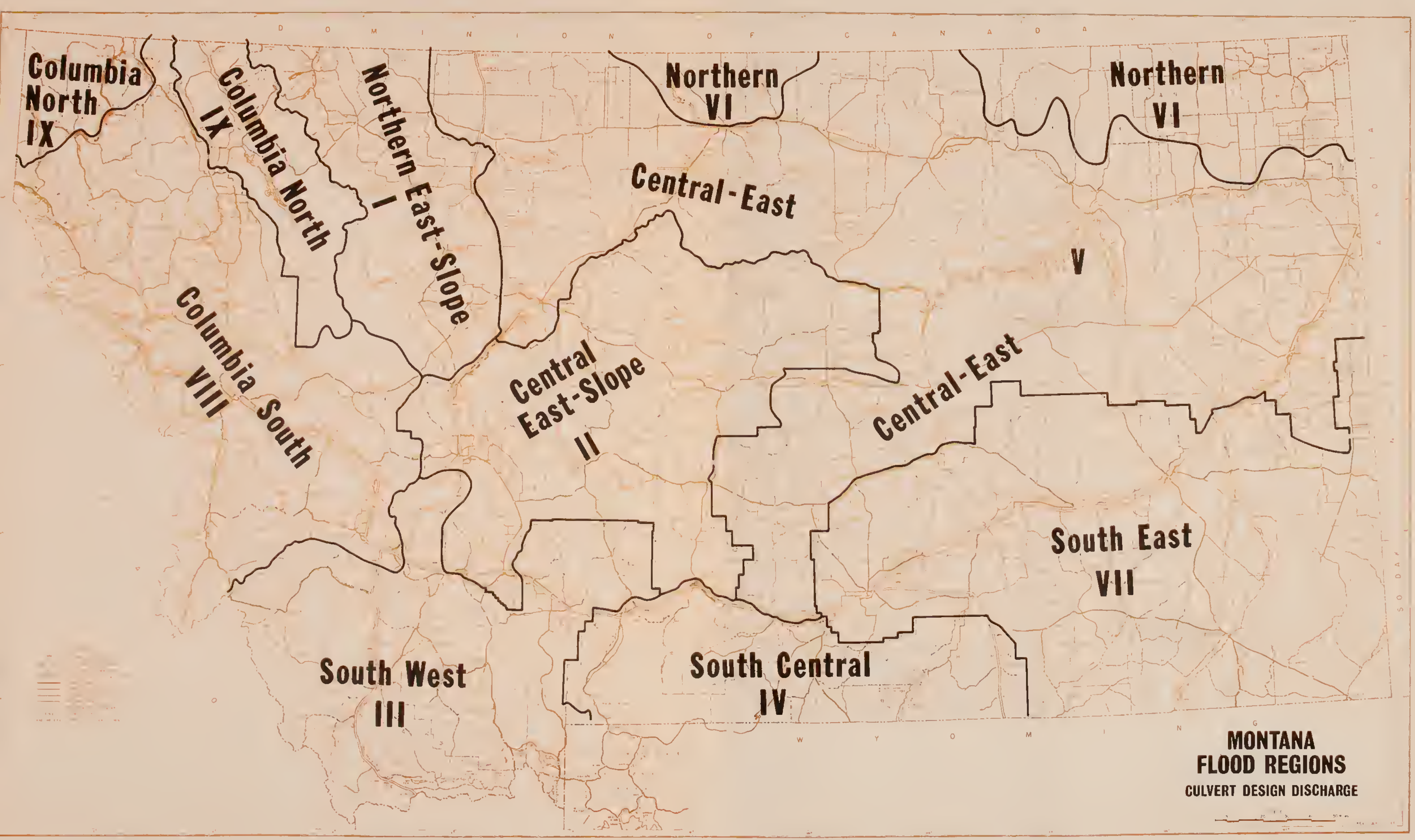
DOMINION OF CANADA
Columbia North IX
Columbia North IX
Northern East-Slope I
Northern VI
Northern VI
Central-East
V
Central East-Slope II
Central-East
Columbia South VIII
South East VII
South West III
South Central IV
WYOMING
MONTANA
FLOOD REGIONS
CULVERT DESIGN DISCHARGE

Fig 5

b_o = regression constant
$b_1 \ldots b_n$ = exponents determined from regression
$X_1 \ldots X_n$ = independent variables

Since linear regression always produces the following type of equation

$$Q = b_o + b_1X_1 + b_2X_2 \ldots . + b_nX_n \tag{17}$$

relations of the form of Eq. (16) are obtained by using the logarithms of all variables. In this study the $\log_{10}$ of all variables was used. The prediction equations resulting from linear regression were then of the form,

$$\log Q = \log b_o + b_1 \log X_1 + b_2 \log X_2 \ldots . + b_n \log X_n \tag{18}$$

Thus an anti-$\log_{10}$ transformation produced the final form of the prediction equation shown in Eq. (16).

It should be realized that the flood values given by prediction equations obtained using the $\log_{10}$ of the data are not the same as the flood values which would be produced by prediction equations obtained using the non-transformed data values. For example, if

$$Q_1 = b_oX_1^{b_1}X_2^{b_2}X_3^{b_3} \tag{19}$$

is the log-linear regression equation resulting from a given set of data, and the regression equation obtained using the non-transformed flows from the same data is given by

$$Q_2 = b_o + b_1X_1 + b_2X_2 + b_3X_3 \tag{20}$$

then for any given values of X_1, X_2, and X_3 it is almost certain that the resulting Q_1 and Q_2 values will not be the same. Because of this it was necessary to choose whether log-linear or simple linear regression best met the objectives of the study. The basis for this decision, which was to use log-linear regression, is as follows:

Simple linear regression develops prediction equations by minimizing the quantity

$$\sum_{i=1}^{N} (Q_A - Q_R)^2 \qquad (21)$$

where Q_A = estimated flood peak for a given recurrence interval
Q_R = corresponding flood peak obtained from regression
N = number of watersheds in the particular region

Therefore simple linear regression also minimizes the standard error of estimate which is:

$$S.E. = \frac{1}{N} \Sigma (Q_A - Q_R)^2 \qquad (22)$$

Log-linear regression, however, develops the prediction equations by minimizing the quantity

$$\sum_{i=1}^{N} \log(Q_R/Q_A)^2 \qquad (23)$$

Experience in this study proved that minimizing this quantity tends to produce lower average percent error values than if simple regression is used. This happens because minimizing the term $\Sigma \log_{10}(Q_R/Q_A)^2$ also tends to produce $Q_R/Q_A \rightarrow 1$, since the logarithm of 1 is zero. This is precisely the result desired if we want to minimize average percent error. These average percent error values are discussed in detail under ERROR ANALYSIS, but basically the average percent error equation is as follows.

$$E = \frac{1}{N} \sum_{i=1}^{N} \frac{(Q_A - Q_R)}{Q_A} \times 100 \qquad (24)$$

Since it is felt that the designer is more interested in minimizing the average percent error produced by a given prediction equation than in minimizing the standard error of the equation, the log-linear regression, which tends to produce lower average percent error values, appears to be the more desirable model for use in this study.

Another and completely independent argument in favor of prediction equations of the form of Eq. (16) is found in the basic hydrological physical interrelationships. Thus if drainage area and a rainfall variable affect flood peaks it seems reasonable that the rainfall variable should produce its effect as a multiplier k_R operating on all the equation to the right of the equal sign such as

$$Q = k_R(b_o A^b) \tag{25}$$

rather than as an additive ΔQ_R term which is a function of the rainfall variable only and completely independent of drainage area size such as

$$Q = b_o A^b + \Delta Q_R \tag{26}$$

In Eq. (26) even if rainfall were zero there would still be a flood, an illogical result.

For the above reasons Eq. (16) appeared to be the desirable form of the model for the prediction equations.

$$Q = b_o X_1^{b_1} X_2^{b_2} \dots X_n^{b_n} \tag{16}$$

However, the question arose as to whether a prediction equation of this form for which the values of the constant b_o and exponents b_1, b_2 ... b_n were determined by minimizing the quantity $\Sigma(Q_A - Q_R)^2$ might be superior to the result obtained from the log-linear regression method which has been described. To determine such a prediction equation required the use of a non-linear regression program.

NON-LINEAR REGRESSION

An alternate set of prediction equations was developed using non-linear regression. This was accomplished using a Least Squares Estimation of Non-Linear Parameters Fortran IV program[18] which accepted the log-linear regression equations, in the form given by (16), and altered (by trial and error) the coefficient and exponents of the independent variables so as to minimize the quantity $\Sigma(Q_A - Q_R)^2$.

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[18] Marquardt, Donald W., April 1965 revision NLIN2, IBM Share Library.

The non-linear regression minimizes the standard error of estimate Eq. (22), as does simple linear regression. Thus the only difference between these two types of regression is that non-linear regression operates using the mathematical model shown in Eq. (16) while Eq. (17) is used as the basic model for simple linear regression.

However, it was found that the non-linear prediction equations were quite inaccurate for predictions of the lower magnitude floods. This resulted because these small magnitude floods were predicted with the same standard error as the relatively large flood events. As a result the percent error values for the smaller floods tended to be extremely high while the larger floods consistently had quite low percent error values. In other words the prediction equations gave undue weight to the large magnitude floods, and paid little attention to floods of small magnitude. Because of this the non-linear regression results were abandoned in favor of the log-linear equations for which the percent error values are fairly constant regardless of flood magnitude.

Table 9 presents the prediction equations developed for each region using log-linear regression. It should be noted that the equations have individually optimized exponents for each independent variable for each recurrence interval. For Regions III, V, and VIII it was necessary to develop alternate equations for use if % Forest falls outside the range of data for the study watersheds.

AVERAGE EXPONENT REGRESSION EQUATIONS

The log-linear regression analysis program described produced 5 different prediction equations for each of 8 regions of the state, and 10 different equations for Region V since the exponents on the independent variables were different for each recurrence interval. This resulted in a total of

TABLE 9

Prediction Equations for Various Recurrence Intervals

$$Q_1 = b_o X_1^{b_1} X_2^{b_2} \dots X_n^{b_n}$$

(Individual Exponent Equations)

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
		--Exponents $b_1 \dots b_n$ for above variables[1] --					
		REGION I					
Q_2	.0673	.920	1.414				
Q_5	.839	.862	.948				
Q_{10}	3.195	.834	.695				
Q_{25}	13.40	.805	.419				
Q_{50}	103.1	.741					
		REGION II					
Q_2	5.483	.724				-.162	2.295
Q_5	16.41	.716				-.293	2.406
Q_{10}	29.31	.709				-.347	2.371
Q_{25}	55.08	.703				-.400	2.284
Q_{50}	83.56	.695				-.427	2.201
		REGION III For % Forest > 19%					
Q_2	.00172	.970	1.390	.758			
Q_5	.00944	.919	1.087	.738			
Q_{10}	.0793	.911		1.131			
Q_{25}	.1549	.877		1.061			
Q_{50}	.2301	.855		1.023			
		Alternate Equations if Forest is < 19%					
Q_2	.00356	.918	2.203				
Q_5	.190	.869	1.879				
Q_{10}	.0450	.843	1.715				
Q_{25}	.115	.811	1.536				
Q_{50}	.207	.791	1.427				
		REGION IV					
Q_2	4.57	1.045					
Q_5	9.46	.973					
Q_{10}	14.13	.932					
Q_{25}	21.98	.885					
Q_{50}	29.44	.854					

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[1] See Table 8 for meaning of variable designations

TABLE 9 (cont.)

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
		--Exponents b_1 ... b_n for above variables--					
REGION V For Forest < 9%							
Q_2	27.04	.510		-.428			
Q_5	60.26	.547		-.471			
Q_{10}	84.62	.565		-.490			
Q_{25}	116.1	.581		-.508			
Q_{50}	138.8	.591		-.519			
Alternate if % Forest > 9%							
Q_2	28.18	.472					
Q_5	62.97	.506					
Q_{10}	88.72	.521					
Q_{25}	122.0	.536					
Q_{50}	146.0	.545					
REGION VI							
Q_2	14.89	.663					
Q_5	42.07	.634					
Q_{10}	59.16	.642					
Q_{25}	87.50	.638					
Q_{50}	109.6	.636					
REGION VII							
Q_2	69.75	.484		-.434			
Q_5	188.8	.443		-.461			
Q_{10}	308.3	.421		-.467			
Q_{25}	509.3	.397		-.469			
Q_{50}	696.6	.382		-.469			
REGION VIII For % Forest > 16%							
Q_2	2.415	1.00	1.708	-1.168			
Q_5	14.32	.975	1.528	-1.307			
Q_{10}	38.46	.961	1.433	-1.398			
Q_{25}	116.9	.948	1.331	-1.509			
Q_{50}	243.8	.940	1.266	-1.591			
Alternate if Forest < 16%							
Q_2	.0852	.945	1.25				
Q_5	.344	.908	1.02				
Q_{10}	.700	.890	.888				
Q_{25}	1.54	.871	.742				
Q_{50}	2.55	.859	.646				
REGION IX							
Q_2	1.064	.937	1.284		-1.099		
Q_5	5.133	.900	1.145		-1.397		
Q_{10}	10.23	.888	1.095		-1.560		
Q_{25}	23.28	.876	1.023		-1.754		
Q_{50}	38.39	.869	.982		-1.880		

50 different equations. This many equations was considered excessive, particularly so because of the desire to prepare graphical solutions for both field and office use. To solve this problem it was decided to try to develop a set of equations with the same average exponents for a given independent variable for a given region for all recurrence intervals to which the equation applied, with only the regression constant varying with the recurrence interval of the flood. If this were successful, a graphical solution could be drawn for any one recurrence interval flood in a given region, and the floods of different recurrence intervals could be found simply by multiplying the flood obtained from the graph by a conversion factor appropriate to other recurrence intervals.

The five recurrence interval equations for any given region contain either 1, 2, or 3 independent variables but all are of the form

$$Q_i = b_{o,i} X_1^{b_{1,i}} X_2^{b_{2,i}} \ldots\ldots X_n^{b_{n,i}} \qquad (27)$$

i = 2,5,10,25,50 year

$X_1 \ldots X_n$ = independent variable variables

$b_{1,i} \ldots b_{n,i}$ = exponents determined from regression

$b_{o,i}$ = regression constants

For example the equations for the case of three independent variables would be as follows.

$$\begin{aligned} Q_2 &= b_{o,2} X_1^{b_{1,2}} X_2^{b_{2,2}} X_3^{b_{3,2}} \\ Q_5 &= b_{o,5} X_1^{b_{1,5}} X_2^{b_{2,5}} X_3^{b_{3,5}} \\ Q_{10} &= b_{o,10} X_1^{b_{1,10}} X_2^{b_{2,10}} X_3^{b_{3,10}} \qquad (28) \\ Q_{25} &= b_{o,25} X_1^{b_{1,25}} X_2^{b_{2,25}} X_3^{b_{3,25}} \\ Q_{50} &= b_{o,50} X_1^{b_{1,50}} X_2^{b_{2,50}} X_3^{b_{3,50}} \end{aligned}$$

The exponents for each of the 10 sets of equations were averaged in the following way.

$$\overline{b}_1 = \frac{1}{5} \sum_{i=2yr}^{50yr} b_{1,i}$$

$$\overline{b}_2 = \frac{1}{5} \sum_{i=2yr}^{50yr} b_{2,i} \tag{29}$$

$$\overline{b}_3 = \frac{1}{5} \sum_{i=2yr}^{50yr} b_{3,i}$$

This procedure produced one set of average exponents denoted by $\overline{b}_1$, $\overline{b}_2$, and $\overline{b}_3$ for each region in place of the previous five sets of individual exponents. Thus the five recurrence interval equations could be written in terms of these average exponents with the only difference being the coefficient $b_{o,i}$ in each equation as shown below.

$$Q_2 = b_{o,2} X_1^{\overline{b}_1} X_2^{\overline{b}_2} X_3^{\overline{b}_3}$$

$$Q_5 = b_{o,5} X_1^{\overline{b}_1} X_2^{\overline{b}_2} X_3^{\overline{b}_3}$$

$$Q_{10} = b_{o,10} X_1^{\overline{b}_1} X_2^{\overline{b}_2} X_3^{\overline{b}_3} \tag{30}$$

$$Q_{25} = b_{o,25} X_1^{\overline{b}_1} X_2^{\overline{b}_2} X_3^{\overline{b}_3}$$

$$Q_{50} = b_{o,50} X_1^{\overline{b}_1} X_2^{\overline{b}_2} X_3^{\overline{b}_3}$$

Calling Q values calculated from Eqs. (30) Q_i and the expected flood magnitude Q_A, we may now plot Fig. 6 for each recurrence interval.

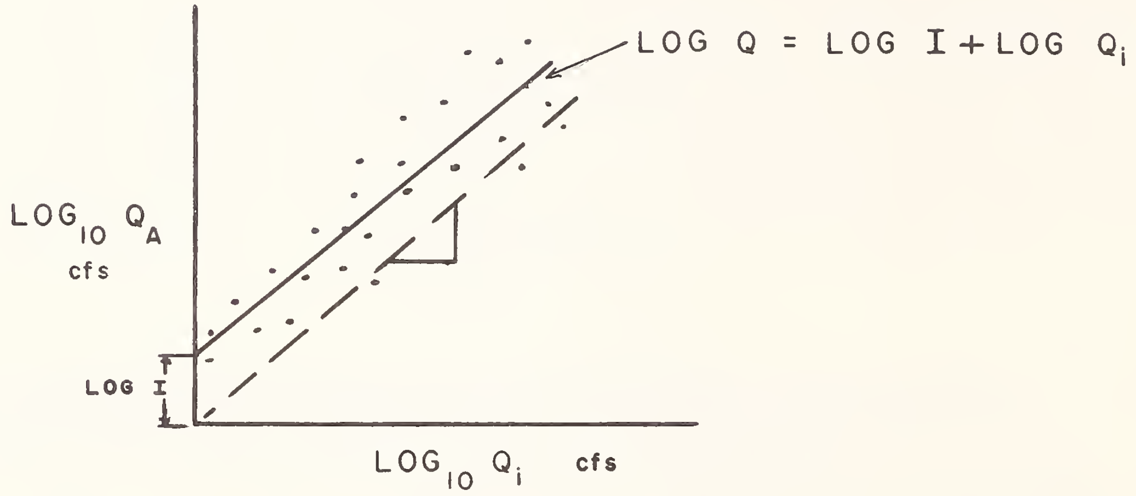

Fig. 6 Plot of Estimated Floods vs Q_i

On this graph a 45° line through the origin simply represents the relation

$$\hat{Q} = Q_i \tag{31}$$

where $\hat{Q}$ is the ordinate read from the line (conventional statistical notation). We would expect that such a line does not minimize the expression

$$\Sigma[\log_{10}(\hat{Q}/Q_A)]^2 \tag{32}$$

We can optimize the fit of the calculated $\hat{Q}$ values however by finding for each recurrence interval the regression constant for Eqs. (30) which does minimize Eq. (32). Changing the regression constant amounts only to shifting the 45° line parallel to itself. From Eqs. (30) it should be clear that we are free only to change the regression constant, since we have already predetermined the average exponents $\overline{b}_1$, $\overline{b}_2$, and $\overline{b}_3$, and the values of X_1, X_2, and X_3 are fixed by watershed and/or hydrologic characteristics of the watersheds, thus determining completely the slope of the line.

A further linear regression is therefore performed, again using logarithms, which results in an optimized prediction equation

$$\text{Log } \hat{Q} = \text{Log } I + \text{Log } Q_i \tag{33a}$$

where log I is the intercept for the line of best fit as shown in Fig. 6. This relation is equivalent to

$$\hat{Q} = I(b_o X_1^{\bar{b}_1} X_2^{\bar{b}_2} X_3^{\bar{b}_3}) = b_i X_1^{\bar{b}_1} X_2^{\bar{b}_2} X_3^{\bar{b}_3} \tag{33}$$

where it should be understood that this equation applies to the particular recurrence interval under consideration. Using subscripts i to denote recurrence intervals and $Q_{\bar{R}}$ to denote flood magnitude calculated by the average exponent regression equations, we can then write the general form of the optimized average exponent prediction equation as

$$Q_{\bar{R}_i} = \hat{Q}_i = b_i X_1^{\bar{b}_1} X_2^{\bar{b}_2} X_3^{\bar{b}_3} \quad (i=2, \text{ .. } 50\text{yr}) \tag{34}$$

Provided these equations predict floods of various recurrence intervals with sufficient accuracy, we can now express any given recurrence interval flood in terms of a conversion factor times any other recurrence interval flood.

The average exponent equations as finally derived for each region are given in Table 10. Again in Regions III, V, and VIII it was necessary to develop alternate prediction equations for use if values of % Forest cover fall outside the range of data for which the equations containing F are considered to apply.

TABLE 10

Prediction Equations for Various Recurrence Intervals

$$Q_{\overline{R}_i} = b_i X_1^{\overline{b}_1} X_2^{\overline{b}_2} X_3^{\overline{b}_3}$$

(Average Exponent Equations)

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
		Exponents $\overline{b}_1$, $\overline{b}_2$ ... $\overline{b}_n$ for above variables					

REGION I

Q_2	.7078	.855	.869				
Q_5	1.165	.855	.869				
Q_{10}	1.500	.855	.869				
Q_{25}	1.953	.855	.869				
Q_{50}	103.0	.7414					

$Q_5 = 1.65\ Q_2$ $Q_{10} = 2.12\ Q_2$ $Q_{25} = 2.76\ Q_2$

REGION II

Q_2	9.354	.710				-.326	2.311
Q_5	19.655	.710				-.326	2.311
Q_{10}	29.031	.710				-.326	2.311
Q_{25}	43.948	.710				-.326	2.311
Q_{50}	57.769	.710				-3.26	2.311

$Q_5 = 2.10\ Q_2$ $Q_{10} = 3.10\ Q_2$ $Q_{25} = 4.70\ Q_2$ $Q_{50} = 6.18\ Q_2$

REGION III

For % Forest > 19%

Q_2	.00325	.944	1.239	.748			
Q_5	.00500	.944	1.239	.748			
Q_{10}	.0932	.900		1.103			
Q_{25}	.1178	.900		1.103			
Q_{50}	.1367	.900		1.103			

$Q_5 = 1.54\ Q_2$ $Q_{25} = 1.25\ Q_{10}$ $Q_{50} = 1.47\ Q_{10}$

Alternate if % Forest < 19%

Q_2	.00432	.846	1.752				
Q_5	.00662	.846	1.752				
Q_{10}	.00829	.846	1.752				
Q_{25}	.0105	.846	1.752				
Q_{50}	.0121	.846	1.752				

$Q_5 = 1.53\ Q_2$ $Q_{10} = 1.92\ Q_2$ $Q_{25} = 2.43\ Q_2$ $Q_{50} = 2.81\ Q_2$

TABLE 10 (cont.)

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
		Exponents $\bar{b}_1, \bar{b}_2, \ldots \bar{b}_n$ for above variables					

REGION IV

Q_2	8.455	.938					
Q_5	11.596	.938					
Q_{10}	13.667	.938					
Q_{25}	16.272	.938					
Q_{50}	18.209	.938					

$Q_5 = 1.37 Q_2$ $Q_{10} = 1.62 Q_2$ $Q_{25} = 1.92 Q_2$ $Q_{50} = 2.15 Q_2$

REGION V
For % Forest < 9%

Q_2	23.102	.559		-.483			
Q_5	57.978	.559		-.483			
Q_{10}	92.663	.559		-.483			
Q_{25}	124.776	.559		-.483			
Q_{50}	154.017	.559		-.483			

$Q_5 = 2.50 Q_2$ $Q_{10} = 4.01 Q_2$ $Q_{25} = 5.39 Q_2$ $Q_{50} = 6.65 Q_2$

Alternate if % Forest > 9%

Q_2	24.074	.5161					
Q_5	60.775	.5161					
Q_{10}	90.442	.5161					
Q_{25}	130.796	.5161					
Q_{50}	161.461	.5161					

$Q_5 = 2.52 Q_2$ $Q_{10} = 3.75 Q_2$ $Q_{25} = 5.41 Q_2$ $Q_{50} = 6.69 Q_2$

REGION VI

Q_2	18.279	.643					
Q_5	47.340	.643					
Q_{10}	65.501	.643					
Q_{25}	96.250	.643					
Q_{50}	121.591	.643					

$Q_5 = 2.58 Q_2$ $Q_{10} = 3.57 Q_2$ $Q_{25} = 5.26 Q_2$ $Q_{50} = 6.65 Q_2$

REGION VII

Q_2	87.451	.425		-.460			
Q_5	200.343	.425		-.460			
Q_{10}	301.692	.425		-.460			
Q_{25}	458.716	.425		-.460			
Q_{50}	595.528	.425		-.460			

$Q_5 = 2.29 Q_2$ $Q_{10} = 3.44 Q_2$ $Q_{25} = 5.25 Q_2$ $Q_{50} = 6.82 Q_2$

TABLE 10 (cont.)

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
		Exponents $\bar{b}_1, \bar{b}_2, \ldots \bar{b}_n$ for above variables					

REGION VIII
For % Forest > 16%

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
Q_2	18.728	.966	1.453	-1.394			
Q_5	28.421	.966	1.453	-1.394			
Q_{10}	34.706	.966	1.453	-1.394			
Q_{25}	42.437	.966	1.453	-1.394			
Q_{50}	47.957	.966	1.453	-1.394			

$Q_5 = 1.52\ Q_2$ $\quad$ $Q_{10} = 1.86\ Q_2$ $\quad$ $Q_{25} = 2.26\ Q_2$ $\quad$ $Q_{50} = 2.56\ Q_2$

Alternate if % Forest < 16%

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
Q_2	.345	.895	.909				
Q_5	.523	.895	.909				
Q_{10}	.639	.895	.909				
Q_{25}	.781	.895	.909				
Q_{50}	.883	.895	.909				

$Q_5 = 1.5\ Q_2$ $\quad$ $Q_{10} = 1.85\ Q_2$ $\quad$ $Q_{25} = 2.27\ Q_2$ $\quad$ $Q_{50} = 2.56\ Q_2$

REGION IX

Flood	Regression Constant b_o	A	PS	F	SI	%>6000	I
Q_2	5.977	.894	1.106		-1.532		
Q_5	8.340	.894	1.106		-1.532		
Q_{10}	9.633	.894	1.106		-1.532		
Q_{25}	11.068	.894	1.106		-1.532		
Q_{50}	12.036	.894	1.106		-1.532		

$Q_5 = 1.39\ Q_2$ $\quad$ $Q_{10} = 1.61\ Q_2$ $\quad$ $Q_{25} = 1.85\ Q_2$ $\quad$ $Q_{50} = 2.02\ Q_2$

ERROR ANALYSIS

After obtaining the results from the regression analysis it was evident that some method of assessing the accuracy of the regression equations was needed. To accomplish this an error analysis program was written which utilized percent error as the means of judging the accuracy of the prediction equations for each region in Montana. Here, percent error is defined as

$$e_i = \frac{(Q_A - Q_R)}{Q_A} \times 100 \qquad (35)$$

where e_i = percent error for a particular watershed
Q_A = actual flood for a specific recurrence interval
Q_R = predicted flood for some recurrence interval

Similarily, E is defined as the average percent error in the region for the recurrence interval under consideration by

$$E = \frac{1}{N} \sum_{i=1}^{N} e_i \qquad (36)$$

Average Percent Error

An error analysis program was developed which calculated percent error values using both individual exponent and average exponent equations for every gaging station in each region. The program also determined an average percent error value for each of the five recurrence interval equations in every region. This was done by taking the numerical average of the sum of the absolute values of all individual percent error values for the given flood frequency and region as indicated by Eq. (36). A comparison of these average percent errors as found for the individual exponent equations and the average exponent equations reveals the accuracy loss (if any) incurred by using the average exponent equations.

A summary of the average percent error values is given in Table 11. We note that in all nine regions the values for the average exponent equations are slightly larger or equal to the values for the individual exponent equation as would be expected. However for the 10, 25, and 50 year floods the difference is generally not significant. Therefore, in most regions of Montana there is little accuracy loss produced by using the average rather than the individual exponent equations. Based on this finding it was decided to use the average exponent equations for the construction of graphical solutions to the prediction equations.

It is noteworthy that the average percent errors in Table 11 are approximately the same for all recurrence intervals in a given region. However, there is considerable variation from one region to another. The smallest errors occur for Regions IX, III and I amounting to about 20-30%.

In some regions, such as Region V, the percent error values tend to be slightly higher for the 2-yr than for the 10, 25, and 50 year floods, while intuitively it would seem that these error values should be smaller for the lower recurrence intervals. However, in interpreting the meaning of the average percent error values, we note that they are only a measure of the accuracy of the regression equations in predicting the estimated flood values for a particular region. Therefore the average percent error values are not a measure of the real accuracy with which the estimated flood values themselves are determined. Recall that the estimated flood values for each recurrence interval were determined by the Regional Frequency Analysis and no attempt was made to assess their accuracy. In short, the average percent error values represent a measure of how well the regression equations fit the input data which consisted of the estimated recurrence interval floods (dependent variable), and the measured watershed parameters (independent variables).

Graphical solutions were constructed for the predicted Q_2 floods for each of the nine regions. The appropriate conversion factors were calculated and tabulated to allow the designer to obtain floods for other recurrence intervals. For example,

TABLE 11

Average Percent Error Values
Flood Magnitude Q_A vs Regression Equation Prediction Value Q_R

$$E = \frac{1}{N} \Sigma \quad e_i$$

EQUATION	Q_2	Q_5	Q_{10}	Q_{25}	Q_{50}
REGION I Northeast Slope					
Individual Exponent	25%	19	20	22	27%
Average Exponent	38%	20	22	32	28%
REGION II Central East Slope					
Individual Exponent	44%	38	37	38	45%
Average Exponent	55%	40	38	44	51%
REGION III Southwest					
Individual Exponent	18%	20	27	29	30%
Average Exponent	19%	20	27	28	29%
REGION IV South-Central					
Individual Exponent	37%	32	30	31	34%
Average Exponent	41%	33	30	32	34%
REGION V Central-East					
Individual Exponent	57%	48	48	50	53%
Average Exponent	57%	48	51	49	52%
REGION VI Northern					
Individual Exponent	33%	33	35	37	39%
Average Exponent	40%	43	40	43	47%
REGION VII Southeast					
Individual Exponent	60%	53	52	53	56%
Average Exponent	69%	54	52	52	56%
REGION VIII Columbia-South					
Individual Exponent	43%	42	43	45	57%
Average Exponent	50%	43	43	45	47%
REGION IX Columbia-North					
Individual Exponent	16%	19	21	22	23%
Average Exponent	22%	19	21	22	24%

$$Q_{\bar{R}_i} = \frac{b_i}{b_2} Q_{\bar{R}_2} \tag{37a}$$

When the number of independent variables in the prediction equation changed, as for Q_{50} in Region I, Q_{10}-Q_{50} in Region III, and for Region V when $F > 9\%$, relations for the new lower recurrence intervals were developed, and a new set of multipliers determined. In this case $Q_{\bar{R}_i}$ values for the higher recurrence intervals may be calculated from the floods for the lower recurrence interval, $Q_{\bar{R}_L}$ by

$$Q_{\bar{R}_i} = \frac{b_i}{b_L} Q_{\bar{R}_L} \tag{37b}$$

These design charts together with complete instructions for their use are presented in Appendix B.

Confidence Intervals

After the average exponent equations were obtained, confidence intervals were constructed to visually aid in defining the accuracy of the equations in a probabilistic manner. Graphs showing these intervals for all 9 regions in Montana are listed in Appendix C.

Confidence intervals provide another (in addition to average percent error) estimate of the reliability of the prediction equations. They express this reliability by designating the probability that the value Q_A of a flood lies within a specified range of the value predicted for that particular recurrence interval and watershed, denoted by $Q_{\bar{R}}$. Fig. 7, for example, shows the 80% and 99% confidence intervals for a particular prediction equation.

Assume that for a particular watershed the predicted flood, $Q_{\bar{R}}$, is found to be 400 cfs. Then referring to Fig. 7 the confidence intervals indicate that we are 80% confident that the value of Q_A will be in the range between about 250 and 550 cfs, and similarly, we have 99% confidence that Q_A is between about 200 and 600 cfs.

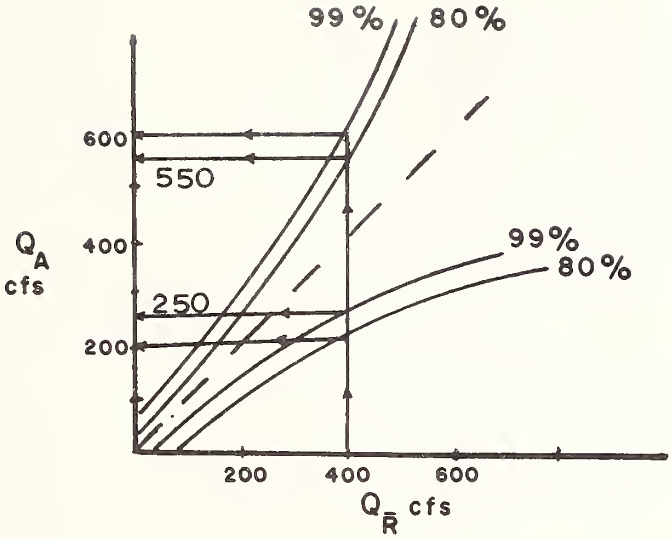

Fig. 7 Procedure for use of Confidence Intervals

The equation used in the construction of these confidence limits is

$$Q_A = Q_{\bar{R}} \pm t(v,1-\alpha/2) \quad 1 + \left[\frac{1}{N} + \frac{(Q_{\bar{R}} - \bar{Q})^2}{\sum\limits_{i=1}^{N} (Q_{\bar{R}_i} - \bar{Q})^2} \right]^{1/2} \tag{38}$$

where $t(v,1-\alpha/2)$ = the t statistic for v degrees of freedom and the $(1-\alpha/2)$ confidence level

N = number of stations in the region

$Q_{\bar{R}}$ = flood peak calculated from prediction equation

$\bar{Q}$ = average of all $Q_{\bar{R}}$ in the region

α = standard error of estimate of all $Q_{\bar{R}}$ values for the region

The primary requirement for the use of this equation is that the variance of the fitted data be essentially constant along the regression line. Fig. 8 reveals that the variance about the 45° line representing $Q_A = Q_{\bar{R}}$ is greater for the larger magnitude floods.

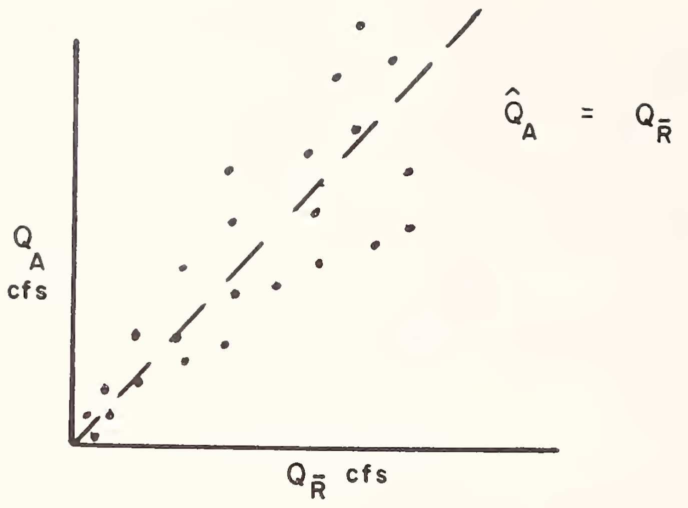

Fig. 8 Typical Arithmetic Plot of Q_A vs $Q_{\bar{R}}$

When the $\log_{10}$ of the Q_A and $Q_{\bar{R}}$ values are plotted, however, the variance appears to be essentially constant as illustrated in Fig. 9.

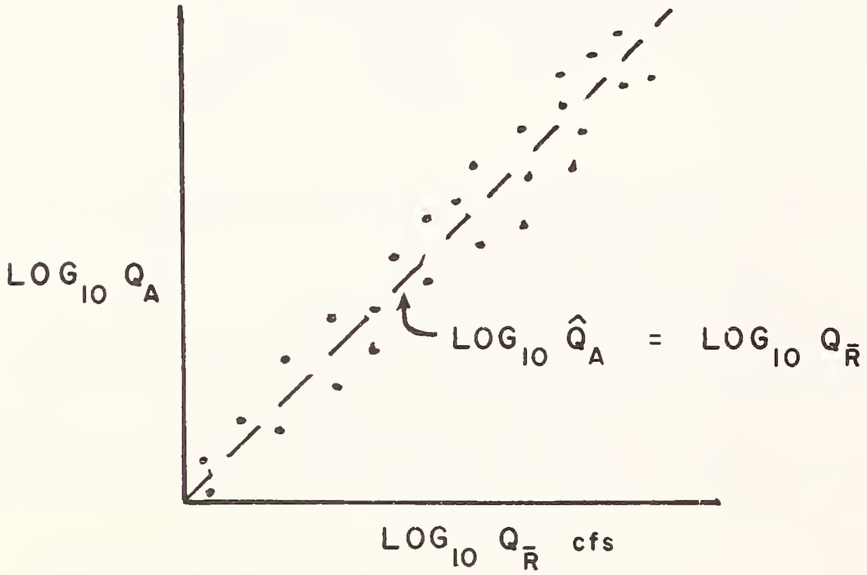

Fig. 9 Typical Log Log Plot of Q_A vs $Q_{\bar{R}}$

Consequently the standard error (S.E.), 80%, 90%, and 99% confidence interval charts were prepared for each Flood Region using the $\log_{10}$ values of Q_A and $Q_{\bar{R}}$. It should be noted that on a log-log plot these confidence limits take the shape of curved lines symmetric about the regression line as shown in Fig. 10.

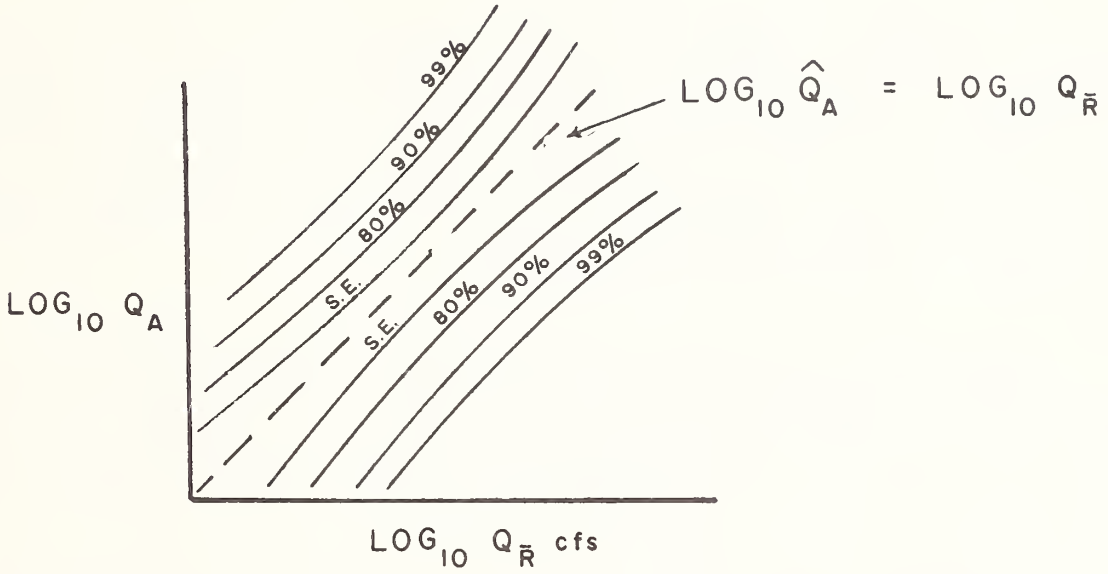
99%
90%
80%
S.E.
S.E.
80%
90%
99%
$LOG_{10}\ \hat{Q}_A = LOG_{10}\ Q_{\bar{R}}$
$LOG_{10}\ Q_A$
$LOG_{10}\ Q_{\bar{R}}$ cfs

Fig. 10 Confidence Intervals on Log Log Plot of Q_A vs $Q_{\bar{R}}$

However when these same confidence lines are transposed to an arithmetic plot they then appear as diverging envelopes as shown in Fig. 11.

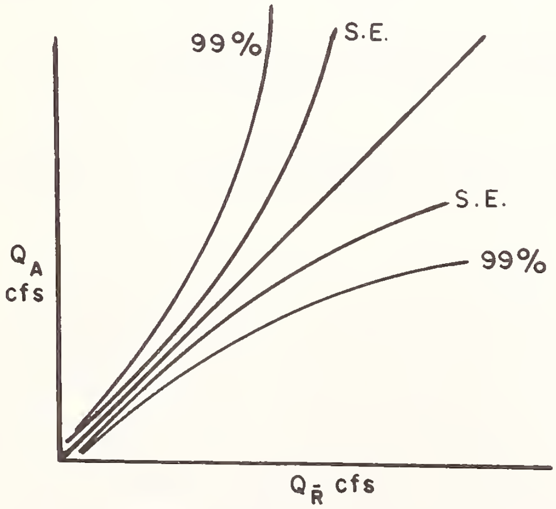

Fig. 11 Confidence Intervals on Arithmetic Plot of Q_A vs $Q_{\bar{R}}$

The divergence of the confidence limits when plotted on arithmetic scales does show an increase of the variance with larger magnitude floods, a result which seems intuitively reasonable.

Confidence interval charts were also plotted for each of the 45 individual exponent equations which have been presented in Table 9 (omitting alternate equations for Regions III, V, and VIII). These charts are on file but are not published as part of this report. They show essentially the same information as the charts presented in Appendix C, except for minor variation between the charts for each recurrence interval.

There are certain instances when the designer may wish to increase or decrease the prediction equation flood because of physical characteristics which are unique to the particular watershed. In this situation, the designer may find the confidence intervals for the particular region helpful in determining by what amount the predicted flood should be altered. For example if a field investigation indicates that a particular watershed has an unusually high (or low) flood potential in relation to most other watersheds in the region, the designer may wish to alter the predicted flood peak accordingly. For this type of decision the confidence intervals could be used as a guide to how much the predicted flood value should be raised or lowered to correspond to a particular probabilistic degree of confidence.

GENERAL EVALUATION RELIABILITY PREDICTION EQUATIONS

The reliability of the prediction equations depends upon the answers to these questions:

1. How accurately did the Regional Analysis define the flood values for each watershed for the various recurrence intervals?
2. How well do these flood values for the various watersheds in a given Flood Region define such flood values for any drainage area in the region for which a culvert may need to be designed?
3. How well do the selected prediction equations fit the flood values for the various watersheds which were obtained from the Regional Analysis?

The discussion of Average Percent Error and Confidence Intervals has hopefully answered question 3 above. Answers to the first two questions can only be arrived at through more subjective judgments and with much less certainty.

The concept of the Regional Analysis as used to extend the effective length of record and to estimate missing flows from records appears eminently sound. It is felt that the estimates of flood values for various recurrence intervals which were obtained for each watershed by this method are much more reliable than those obtained from an analysis of the recorded flows at each station alone. The Regional Analysis method as used in the study tends to average flood conditions between adjacent areas, simulating natural events from a climatological, hydrological, geological (including soils), topographical, and land use and condition standpoint.

The equations for flood prediction which have been developed in this study are probably valid for all drainage areas in Montana within the range of about 1 to 1000 sq miles. Table 12 shows the actual range of drainage areas in each region from which the regression equations were derived.

Table 12

Range Drainage Area Size by Regions

REGION	NAME	DRAINAGE AREA IN SQ MILES	
		MIN.	MAX.
I	Northeast Slope	3.5	609.
II	Central East Slope	0.2	684.
III	Southwest	2.3	2476.
IV	South-Central	50.4	2623.
V	Central-East	0.6	2554.
VI	Northern	7.1	3174.
VII	Southeast	0.1	1974.
VIII	Columbia-South	3.4	2290.
IX	Columbia-North	0.1	4464.

A basic precept of applied regression analysis is that the prediction equations are highly suspect if used beyond the range of data from which they were derived. Except for Region IV, where the minimum drainage area for which data was available was 50.4 sq miles, it is believed that the range of drainage area used justified the confidence in the prediction equations stated above. The prediction equations for Region IV are all obtained from the 2-year recurrence interval equation,

$$Q_2 = 8.44\ A^{.938} \tag{39}$$

The variation of flood peak as the .938 power of drainage area appears reasonable for southern Montana.

A regression analysis made with all watersheds in Regions III and IV combined and with A as the only independent variable gave an average exponent of .945, very close to the .938 value. For a reduction in drainage area size from 50 to 1 sq miles the difference produced by these two exponents would only amount to $.02^{.945-.938}$, which is less than 1%. In addition, examination of residuals for the regression runs from which Eq. (39) was derived shows good agreement for the smaller drainage areas. For these reasons it is believed that the prediction equations for Region IV are reasonably valid for areas as small as 1 sq mile, and perhaps smaller. Additional data for smaller drainage areas, will of course permit a more objective evaluation than is now possible.

The percent forest cover for a small drainage area may vary outside the limits of F which were available for the watersheds studied for a given region. Because it is unsafe to extrapolate beyond the limits of F used to derive the prediction equations, in Regions III, V, and VIII where this may occur, an alternate set of prediction equations in which F does not appear were developed and should be used.

In Regions III and VIII the alternate equations should be used if F is less than 19% and 16% respectively. However, because such F values will be rare in these regions, design charts were not constructed for these alternate equations.

In Region V the alternate equations should be used whenever it is found that F is much greater than 9%, the largest value which was available in this Region. Because larger magnitude F values may occur for small areas, a design chart is presented for Region V based on the alternate equations.

It is axiomatic that the results of any analysis will not be more reliable than the data which is analyzed. Based on general hydrologic judgment, it would appear that the prediction of floods with 50-year recurrence intervals, within a desirable accuracy from an economic standpoint, will require continuous flood records of at least the scope of the present U.S. Geological Survey Program in Montana for another 25-50 years. The variability of data is greatest in Flood Regions V and VII, in central, eastern, and southeastern Montana. Region II the Central-east slope, and Region VIII, the Columbia-South are not far behind. It should not be surprising that Region V demonstrates the greatest variability in flood data. Examination of Fig. 5 shows that this Region extends about 400 miles in a westerly direction from the North Dakota border. It also extends irregularly about 230 miles south from the Canadian border. It seems likely if more watersheds were gaged over this region that better prediction equations would result if Region V were subdivided and/or combined with adjacent regions. Further examination of Fig. 5 shows very little coverage with gaged watersheds in the extreme northwest and through most of the northern portion of the region east of Malta. Additional watershed data is needed in these areas to define their flood characteristics.

From a geographic standpoint it would appear that the southern and/or eastern portion of Region V might be found to fit better with the extreme north-western portion of Region VII if additional watershed data were available. There is also some evidence that the extreme southeastern corner of Montana should be a separate Flood Region rather than part of a more extensive region such as Region VII shown in this report. However flood records from more watersheds are needed to establish this. It appears desireable to gage a few watersheds in Region IV which have drainage areas from less than 1 to about 25 sq miles, with several below about 5 sq miles.

One can only conclude that with additional flood data, the prediction equations presented in this report can be continually improved. This should result from a better definition of hydrologically homogeneous flood regions, as well as from a more reliable prediction equation to fit all the hydrologic variables which may affect the flood producing characteristics within a given flood region.

RATIONAL METHOD

Over the years many formulas have been developed for estimating peak rates of discharges from watersheds. Of these the so called rational formula is probably the best known. Although hydrologists have known for at least 40 years that such formulas have little validity for natural watersheds, they are still widely used.

The rational formula is,

$$Q = CiA \tag{40}$$

where

Q = Peak discharge, cfs

C = Coefficient of runoff

i = Rainfall intensity (inch/hr) for a duration equal to the travel time of the watershed (i is customarily read from a rainfall intensity-frequency-duration curve).

A = Drainage area, acres

The units of the rational formula are based on the relation 1 cfs = 1 acre-inch/hr, which is accurate to within 1%.

The rational formula is presently being used by approximately 90% of all state highway departments to predict flood peaks, although ostensibly such use is primarily for small relatively impervious drainage areas.[19] The reasons for the continued use of this formula despite its failure to adequately model the runoff process are evident in its simplicity. Values of i and A are easily and routinely determined. C values can be read from many published tables. Hence calculations of Q from the rational formula can be performed with utter simplicity by practically anyone. It is important that engineers and technicians considering the use of the rational formula understand the full meaning of each term along with the basis of the formula.

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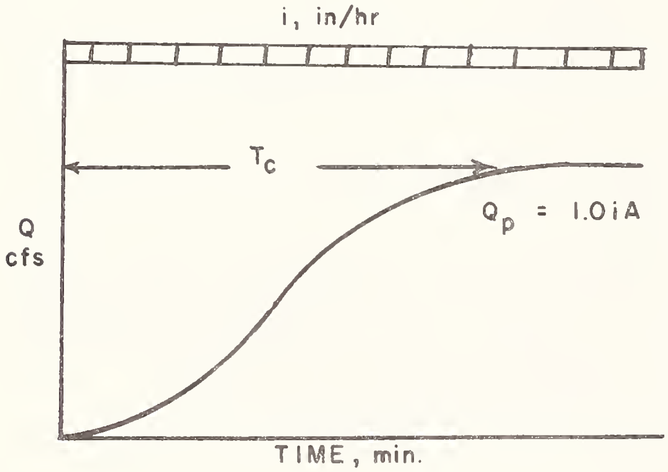
[19] Wycoff, R. L. and T. E. Harbaugh, A Survey of the Hydrologic Design Practices of State Highway Departments, Hydrologic Series Bulletin, Department of Civil Engineering, University of Missouri, Kocla, Missouri.

BASIS FOR RATIONAL FORMULA

The basis for the rational formula is developed by considering the following special case. This involves the analysis of a constant intensity rain falling on an area for which C = 1.0 (actually such rainfalls rarely if ever occur and no such areas exist), for a period of time greater than T_c, assumed here equal to the time required for equilibrium conditions to be established. See TIME OF CONCENTRATION for further discussion of T_c. The resulting runoff hydrograph would be similar to that shown in Fig. 12.



T_c = time to equilibrium, assumed
= time of concentration (min)

Q_p = peak flow (cfs)

A = drainage area (acres)

i = constant rainfall intensity (in/hr)

Fig. 12 Runoff Hydrograph for Constant Intensity Rainfall on Impervious Surface

As indicated the peak flow rate is given by $Q_p = 1.0\ iA$. Furthermore, it should be noted that Q_p is the maximum possible peak flow which will occur if the rainfall rate occurs at the constant rate i in/hr.

It is evident from the hydrograph that Q_p does not occur until the storm duration equals the time when 100% of the drainage area first contributes runoff, which is called the time of concentration of the drainage area, T_c. Thus if the storm duration is less than T_c the flood peak will be less than Q_p.

COEFFICIENT OF RUNOFF

The coefficient of runoff C is a multiplier with value 0.0-1.0 which supposedly indicates the percentage of the rainfall rate i which will appear as surface runoff from the drainage area A. The primary shortcoming associated with the coefficient of runoff is that it neglects the surface and channel storage of rainfall excess required in the runoff process. The use of a constant C value for a particular drainage area assumes that the processes of infiltration, retardation, and evapo-transpiration are constant for all recurrence interval rainstorms for that watershed. Actually it is quite obvious that for most natural watersheds the coefficient of runoff is indeed highly variable and not capable of being accurately predicted. In addition there is no physical reason why the percent of excess rain rate, averaged over the travel time of the watershed, should be directly proportional to the peak rate of flow for the outflow hydrograph. One recent investigation has completely abandoned the pretense that C is a runoff coefficient, and referred to it as complex stochastic function which maps rainfall rates into runoff rates.[20]

However, in the case of small, relatively impervious areas, such as paved surfaces, it becomes possible to predict C with greater reliability. Also, because of the short travel time and limited surface and channel storage involved, equating the average rate of excess water supply to the peak rate of discharge may be more realistic. In these cases C will probably have an average value in the range .85-.95, since it will be true that all the rainfall on a highly impervious area will appear as runoff - except that required to wet the surface and fill depression and channel storage.

INTENSITY OF RAINFALL

In using the rational formula it is always assumed that the design rainfall intensity is read from the intensity-duration-frequency chart at the duration corresponding to T_c, the time of concentration of the drainage area. There is, of course, no reason to believe that peak floods are produced by rain storms of constant intensity. The rainfall intensity read from the chart actually represents the maximum rainfall rate averaged over the time period T_c which occurs during any such time period of a rainstorm of recurrence interval T_p years.

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[20] Schaake, John C. Jr., A Summary of the John Hopkins Drainage Research Project, Vol II, the Progress of Hydrology, Department Civil Engineering, University of Illinois, Urbana, Illinois.

By recurrence interval we mean the average number of years which from a probablity standpoint may be expected to occur to the first storm with an intensity equal to or greater than that specified. In civil engineering this is referred to as average return period or simply recurrence interval[21]. The magnitude of the event, here a rainfall intensity, is often referred to as the T_p year rainstorm or event. The term is generally misunderstood by layman (and unfortunately by others) when it gives the impression that there will be T_p years between such events, when in fact the probability of such an event occurring in any year, which is simply $1/T_p$, remains constant independent of the occurrence of such an event in the previous year or during any recent year.

The time period T_c does not really represent the duration of the rainstorm, or any definite chronological time period in a given rainstorm. It is only a rainfall averaging time period, and for a particular rainstorm the design intensity may occur anywhere during the storm from the start to the cessation of rain.

Although there are several methods of describing time of concentration, in this study T_c is defined as the time required for runoff to travel from the most remote (timewise) point in the drainage to the inlet of the drainage structure. This time will generally consist of a combination of overland flow and channel flow times.

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[21] Benjamin, J. R. and C. A. Cornell, Probability Statistics and Decision for Civil Engineers, McGraw-Hill Book Co., New York, 1970, p 232.

RAINFALL INTENSITY-DURATION-FREQUENCY CURVES

In 1969 the U.S. Weather Bureau-Environmental Science Services Administration (ESSA) published a series of revised storm rainfall maps of Montana for durations from 6 hours to 24 hours and recurrence intervals from 2-years to 100-years[16]. Fig. 13 shows the 2-year recurrence interval 24-hour duration map of this series. From this map one may determine the amount or intensity of storm rainfall to be expected at any point in Montana for the 2-year recurrence interval. The isopluvial lines shown are in tenths of an inch, thus at Billings a storm with total rainfall of 1.4 inches in 24-hours (read on Fig. 13) has a recurrence interval of 2-years which means a probability of occurring in any given year of 1/2. (Probability = 1/recurrence interval.)

From a careful study of the 1969 ESSA maps and U.S. Weather Bureau T.P. 40[22] it was found that intensity-duration data for any station in Montana could be reasonably represented in each of four geographic regions by regional charts such as shown in Fig. 14 for Region I which covers northwestern Montana. On Fig. 13 the heavy dashed lines denote the approximate four regional boundaries.

The family of parallel lines shown on the regional intensity-duration charts illustrated by Fig. 14 are guidelines defining the trend of intensity vs. duration for that region. To determine the 2-year intensity to be expected for any duration one enters Fig. 14 at the 24-hour duration with the 2-year 24-hour intensity in inch/hour calculated by dividing the amount read from Fig. 13 by 24. The 2-year intensity at any desired duration is determined by moving parallel to the curved guidelines and reading the ordinate at this duration. To obtain the rainfall intensity for this duration for the design recurrence interval, one uses the table of multipliers on the chart.

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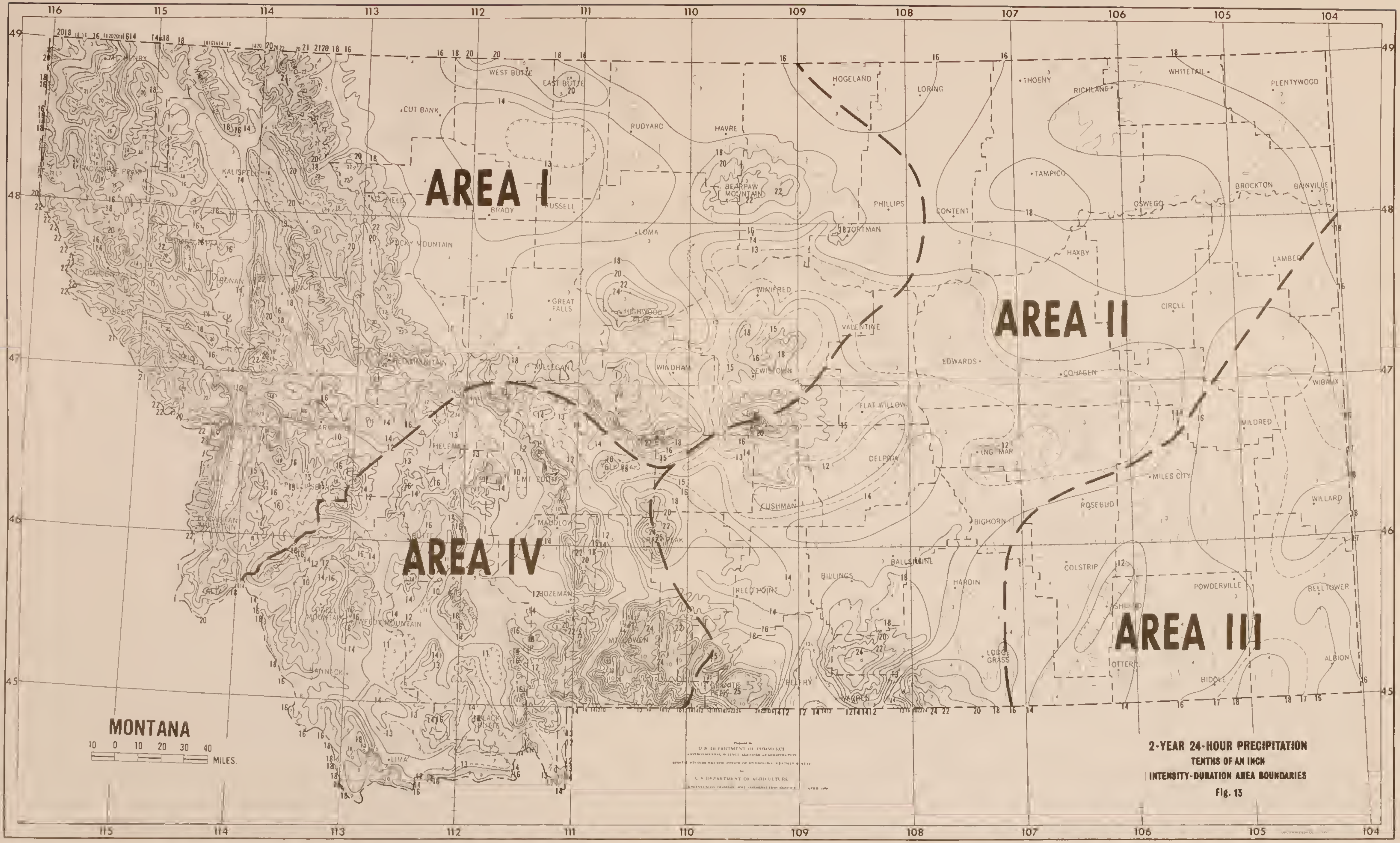
[22] Technical Paper No. 40, Rainfall Frequency Atlas of the U.S., January 1963. Washington, D. C.



2-YEAR 24-HOUR PRECIPITATION
TENTHS OF AN INCH
INTENSITY-DURATION AREA BOUNDARIES
Fig. 13

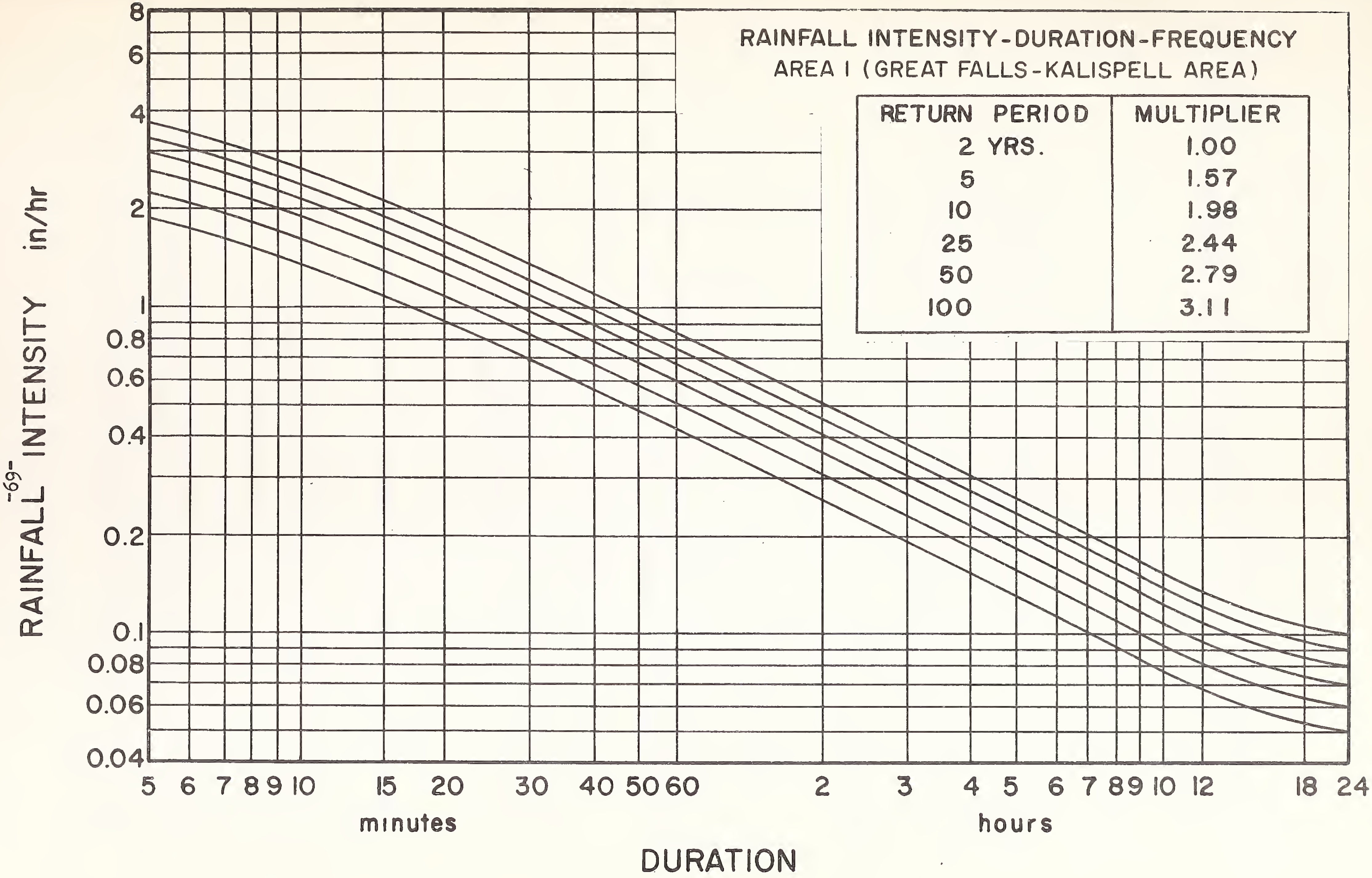


Fig. 14 Rainfall Intensity-Duration-Frequency Chart Area I, Montana

The 2-year 24-hour 1969 ESSA map of Montana and the four regional intensity-duration-frequency charts are presented in Appendix D for ease of use and reference.

TIME OF CONCENTRATION

The standard procedure in the design of hydraulic structures is to assume that the averaging time to be used for rainfall intensity is equal to the time of concentration T_c of the drainage area. Here T_c is taken to mean the largest combination of overland flow time and channel flow time which exists from anywhere in the basin to the outlet or design point. Except for steady state conditions, there is really no logical reasoning to support this assumption.

Steady state conditions rarely, if ever, occur in an actual rainstorm. Hydrologic research in recent years has shown considerable variation associated with T_c for a given watershed. For example, experimental evidence has been collected by Pilgrim[23] which strongly suggests that T_c is not constant but rather is related (nonlinearly) to the magnitude of the flood event itself.

In another recent study Schaake[24] proposes that the duration of the design intensity should in fact be set equal to the numerical average of all observed T_c values for a given drainage area. This represents an attempt to regard T_c as a random variable rather than a constant.

Although studies such as these represent progress toward a better understanding of the rainfall-runoff process, there is little advancement being made toward developing better methods of estimating T_c for ungaged drainages. Therefore, to predict T_c, the designer generally relies upon the methods and assumptions used for many years. The assumption that the rainfall intensity to be used in the rational formula should correspond to a duration

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[23] Pilgrim, D. H., Radioactive Tracing of Storm Runoff on Small Experimental Watersheds, Journal of Hydrology, Vol. 4, 1966, Pg 306.

[24] Schaake, J. C., J. W. Knapp, A Solution to the Surface Runoff Problem, ASCE Journal of the Hydraulics Division, Vol. 93 No. HY6, Nov. 1967, Pg 353.

(averaging time) equal to T_c is still recommended. Recommended empirical methods for estimating T_c are discussed under ESTIMATION DESIGN DISCHARGE BY RATIONAL METHOD in Chapter III.

RECURRENCE INTERVALS FOR RAINFALL AND PEAK FLOW

Another aspect of the rational method concerns the question of the recurrence interval which applies to the calculated value of Q. It is customarily assumed that the predicted peak will have the same recurrence interval as the rainfall intensity from which the flow is calculated. This assumption, like the assumption of a constant runoff coefficient, is quite untrue for anything other than a completely impervious surface.

Fig. 15 illustrates the type of results which can be expected from the rainfall-runoff relationship for a relatively impervious area.

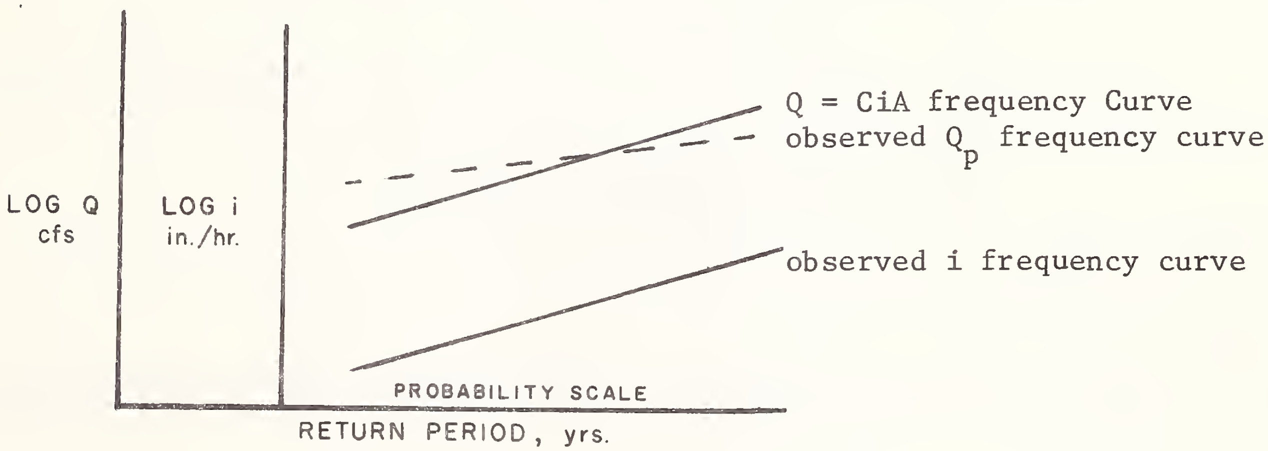

Fig. 15 Frequency Curves for i and Q for Relatively Impervious Area

This sketch shows a comparison between the frequency curve constructed from the observed peak flows and the frequency curve developed from the observed i frequency curve and the rational formula by multiplying ordinates of the i curve by the constant CA. This assumes that corresponding Q and i values have the same recurrence interval. In the case of fairly impervious areas, these two frequency curves will probably be very nearly coincident as shown. Therefore the assumption that corresponding values of Q and i have approximately the same return period is probably not seriously in error.

Fig. 16 displays results that are typical of intense rainstorms and associated peak flows from relatively pervious natural drainages. For such watersheds, the rational formula Q = CiA and the actual flood peak frequency curves will probably not be in close agreement. The discrepancies would be caused by failure of the rational formula to model the rainfall-runoff process correctly, the variability of the runoff coefficient C in different rainstorms, and the irregular time and areal distribution of rainfall.

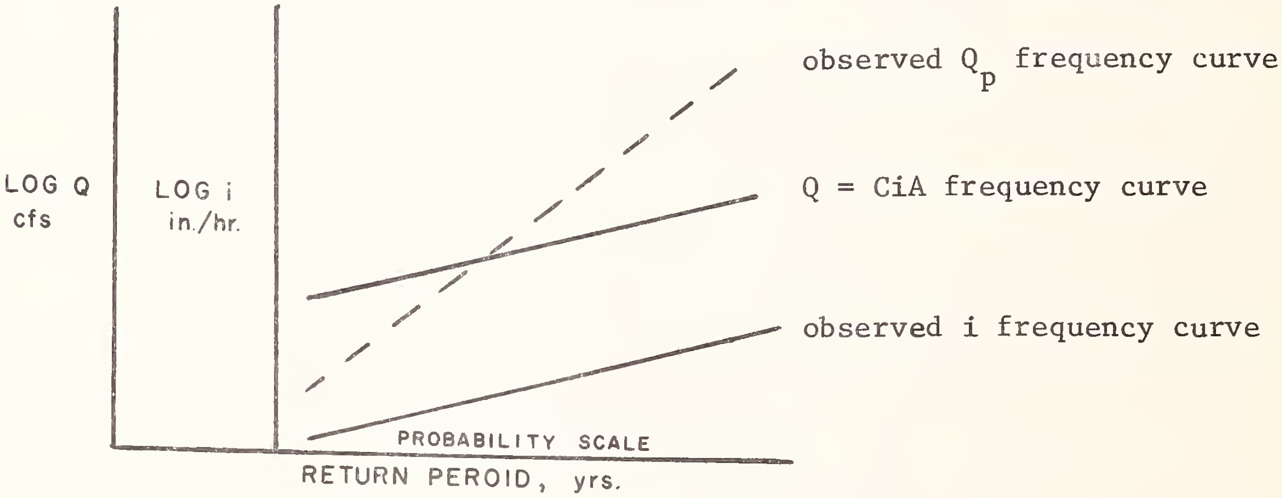

Fig. 16 Frequency Curves for i and Q for Pervious Natural Surface

We conclude, therefore, that when the rational method is applied to a natural watershed it is erroneous to assume that the flood peak and rainstorm intensity values will have the same recurrence interval.

SUMMARY EVALUATION OF RATIONAL FORMULA

From the preceeding discussion it is evident that the application of the rational formula to estimate flood peaks from natural watersheds should be done only as a last resort when other methods cannot be applied. It is important that the designer be aware that the rational formula, even when applied to fairly impervious areas, has virtually no theoretical justification. Moreover, the designer should realize that he must contend with at least four types of uncertainties when applying the rational formula. These are as follows:

1. Uncertainty associated with the selection of the runoff coefficient C.
2. Uncertainty due to the effects of the temporal and areal distribution of the design rainfall intensity.
3. Uncertainty that the actual peak discharge occurs when 100% of the drainage area is contributing runoff at the design point.
4. Uncertainty as to the true recurrence interval of the flood peak which is predicted by the rational formula.

Despite the failure of the rational method to really take into account the nature of the rainfall-runoff process, many engineers still believe with considerable logic, that for relatively impervious areas of small size, (less than 10 acres), it provides a reasonable basis for flood estimates. However, it is not recommended that the rational formula be applied to pervious natural watersheds.

Chapter III

APPLICATION OF RESULTS

FLOOD PREDICTION-NATURAL WATERSHEDS

The result of the stepwise linear regression analysis was the development of individual exponent flood prediction equations for the 2, 5, 10, 25 and 50 year recurrence interval floods for each of 9 contiguous flood regions of Montana. These prediction equations are listed in Table 9, and the boundaries for the flood regions are shown in Fig. 5. The set of flood prediction equations developed as described under AVERAGE EXPONENT REGRESSION EQUATIONS are listed in Table 10. Flood prediction design charts constructed from the latter equations are presented in Appendix B.

FLOOD PREDICTION CHARTS

The flood prediction design charts are based on the average exponent equations previously discussed. The flood magnitude of a given recurrence interval (usually Q_2 denoting T_p = 2 years) is read from the chart. This Q_2 flood (or a flood for any other T_p value) is converted to the 5, 10, 25, or 50-year T_p flood simply by the appropriate recurrence interval multiplier. These multipliers are listed on each design chart.

FLOOD PREDICTION EQUATIONS

The alternative to the use of the design charts for flood peak prediction is the solution of the individual exponent or average exponent regression equations by digital computer, slide rule or logarithmic computation. These equations are listed in Tables 9 and 10. The individual exponent equations probably give somewhat more reliable predictions, although the gain in reliability over the average exponent equations is generally negligible.

PROCEDURE FOR FLOOD PEAK PREDICTION

The steps required in predicting the flood peaks for any desired recurrence interval (T_p up to 50 years) are as follows:

1. Determine the Flood Region, I-IX, in which the design watershed is located.
2. Determine the numerical values for the independent variables required in the prediction equations for the particular region.
3. Use either the individual exponent prediction equations, the average exponent equations, or the design charts to estimate the design flood for the desired recurrence intervals.

Although the designer has a choice between the design charts and the regression equations, it is suggested that the design charts be used. Unless the computations are to be done by digital computer, the charts are more convenient and greatly reduce the chance of computation error.

ESTIMATING WATERSHED PARAMETERS

A total of six watershed parameters may be involved in the use of the design charts and prediction equations, although not more than three are used in any given flood region. They are:

1. A = Drainage area, sq mi
2. F = Percent Forest Cover, %
3. %>6000 = Percent area greater than 6000 ft, %
4. PS = SCS Mean Annual Precipitation, inch
5. I = Precipitation Intensity (2 year-24 hour), inch/24-hr
6. SI = Soil Storage Index, inch

The parameters may be estimated for any given design watershed by the following methods. Unless stated otherwise, these methods were used in this study, although for the watersheds studied, all parameters expect some Soil

Index values and all SCS mean annual precipitation values were obtained directly from a U.S. Geological Survey report.[25]

DRAINAGE AREA

The drainage area A, may be determined from any suitable topographic map on which it is possible to designate the boundaries of the watershed. However for this study drainage area was determined from the latest U. S. Geological Survey Surface Water Records reports.

PERCENT AREA ABOVE 6000 FT

The percentage of the drainage area above 6000 ft, %>6000, may be found from 1:250,000 Army Map Service maps (or any contour map) by laying a rectangular grid over the map, counting the number of grid intersections for all parts of the drainage area above 6000 ft, and then dividing this count by the total number of grid intersections which are required to cover the area.

PERCENT FOREST COVER

The grid method used to find %>6000 can also be used to estimate F, the percentage of the drainage area covered by forest on a map which designates forest areas. The 1:250,000 Army Map Service maps indicate forested regions in green color, and were used in this study to obtain F values. For small drainage areas recent aerial photo mosaics may provide better data than the Army Service Maps.

SCS MEAN ANNUAL PRECIPITATION

Mean annual precipitation PS, is determined from isohyetal maps of Montana prepared for this report from a working copy loaned by the Bozeman SCS State Office. This map is presented in Appendix B.

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[25] Boner, F. C., G. W. Buswell, A Proposed Streamflow-Data Program for Montana, open file report U.S. Geological Survey, 1970, Helena, Montana.

PRECIPITATION INTENSITY (2-year 24-hour)

The precipitation intensity I refers to the 2-year 24-hour duration rainfall amount which is to be expected for the design watershed. The value is referred to as precipitation-intensity rather than amount since for the specified 24-hour duration, it does measure an average intensity. These values may be obtained from the 1969 ESSA Weather Bureau map of Montana shown in Fig. 13 under Rational Method and also presented in Appendix D.

SOIL INDEX

The Soil Index SI, for a drainage area is a concept developed by the U.S. Soil Conservation Service. The SI value is a function of the weighted curve number CN, as given by the relation

$$SI = \frac{1000}{CN} - 10 \qquad (41)$$

This equation gives values of SI ranging from 0 for CN = 100 to 10.0 for CN = 50 and approaching infinity for extremely small values of CN. Most natural watersheds in Montana will have SI ranging from a low of about 2.0 to a high of about 9.0.

To determine the weighted curve number for the watershed the drainage area is classified as to (1) percent in various classifications of land use or cover (with condition rated as poor, medium, or good) and (2) percent of soil classified in the four broad hydrologic classes A, B, C, or D as used by the Soil Conservation Service.

Table 13 lists values of CN for each land use or cover condition on each type of soil for the intermediate soil moisture condition II.

An example will illustrate the procedure. Suppose we have the following data for a given watershed:

TABLE 13

Values of Curve Number for Various Cover-Soil Groupings for Moisture Condition II

Land Use or Cover	Treatment	Hydrologic Condition	Hydrologic Soil Group: A	B	C	D
Fallow	St. Row	--	77	86	91	94
Row Crops	"	Poor	72	81	88	91
	"	Good	67	78	85	89
	Contoured	Poor	70	79	84	88
	"	Good	65	75	82	86
	C & T*	Poor	66	74	80	82
	C & T*	Good	62	71	78	81
Small Grain	St. Row	Poor	65	76	84	88
		Good	63	75	83	87
	Contoured	Poor	63	74	82	85
		Good	61	73	81	84
	C & T	Poor	61	72	79	82
		Good	59	70	78	81
Close-seeded legumes[1/]	St. Row	Poor	66	77	85	89
	"	Good	58	72	81	85
or	Contoured	Poor	64	75	83	85
rotation	"	Good	55	69	78	83
meadow	C & T*	Poor	63	73	80	83
	"	Good	51	67	76	80
Pasture or range		Poor	68	79	86	89
		Fair	49	69	79	84
		Good	39	61	74	80
	Contoured	Poor	47	67	81	88
	"	Fair	25	59	75	79
	"	Good	6	35	70	79
Meadow (Permanent)		Good	30	58	71	78
Woods (Farm Woodlots)		Poor	45	66	77	83
		Fair	36	60	73	79
		Good	25	55	70	77
Farmsteads		--	59	74	82	86
Roads (dirt)[2/]		--	72	82	87	89
(hard surface)[2/]		--	74	84	90	92

[1/] Close-drilled or broadcast

[2/] Including right-of-way

* Contoured and Terraced

Example Soil Index Calculation

Use or Cover	Condition	Soil	% Area	CN	weighted CN	
Fallow		A	15	77	11.5	
Small Grain St. Row	Good	A	15	63	9.4	
Pasture	Good	A	10	39	3.9	
Range	Fair	B	20	69	13.8	
Meadow	Good	C	10	71	7.1	
Forest	Poor	D	30	83	24.9	
			100		70.6	say 71

The weighted CN value for the watershed calculated in the last 2 columns is estimated as 71 as shown above. The SI value for this watershed is from Eq. (41)

$$SI = \frac{1000}{71} - 10 = 14.1 - 10 = 4.1$$

Mr. Ron Moshier, Soil Scientist for the Montana U.S. Soil Conservation Service at Bozeman determined (estimated in most cases) the percent soil in the four SCS hydrologic groups for each study watershed. From his soil classifications Mr. Robert Dean, Hydraulic Engineer, U.S. Soil Conservation Service, calculated weighted CN and SI values. These values are tabulated in Appendix A under Col. 18, S-IDX. To estimate values of SI for a particular watershed, either the detailed procedure just described may be accomplished or as an approximation (which may be sufficiently accurate in most cases) an estimate may be made by comparison with the SI values shown for nearby watersheds in Appendix A.

ESTIMATION OF DESIGN DISCHARGE BY RATIONAL FORMULA

A discussion of the rational method of flood peak prediction for small relatively impervious areas by the formula

$$Q = CiA \tag{40}$$

has been presented earlier in this report. This discussion covers individual terms in the formula in detail, and points out the reasons for limiting the use of the rational formula to small, relatively impervious areas. It is strongly recommended that the designer become familiar with this discussion so that he can develop an awareness of the shortcomings and limitations of the so-called rational method for calculating flood peaks.

Typical examples of areas to which the rational formula might be applied are highway pavements and shoulders, highway medians, and urban storm sewer design.

ASSUMPTIONS NECESSARY FOR APPLICATION OF RATIONAL FORMULA

There are three major assumptions which are made in the application of the rational formula:

1. The coefficient of runoff C is known for the rainfall intensity associated with each recurrence interval for a given watershed.
2. The design rainfall intensity i is the value obtained from an intensity-duration curve for the desired rainfall recurrence interval, at a duration corresponding to the time of concentration for the watershed.
3. The peak flow has the same recurrence interval as the design rainfall intensity.

DESIGN PROCEDURE

To use the rational formula to predict the peak runoff the designer should follow the procedure outlined below.

1. Determine the following drainage area parameters:

    a. Drainage area A, acres
    b. Length of overland flow L_o, ft
    c. Length of channel flow L_c, ft
    d. Channel slope S_c, ft/ft
    e. Average slope of overland flow surface S_o, ft/ft

2. From the characteristics of the drainage area estimate the coefficient of runoff from the tabulation presented in Appendix D.

3. Determine the time of concentration T_c for the drainage area and the design rainfall intensity i in inch/hr for the desired recurrence interval. Rainfall intensity is read from the Intensity-Duration-Frequency chart (Appendix B) for the region in which the design area is located. The duration at which i is read on the chart is equal to the time of concentration T_c of the drainage area. T_c is assumed to be the overland flow time plus channel flow time for the portion of the drainage area most remote (timewise) from the design point. Since velocity of flow depends upon Q, which in turn depends on i, an iterative (successive approximation) solution is necessary. A trial value of i may be assumed, from which approximate Q and overland channel flow times are calculated. With an approximate value of T_c thus determined, a revised value of i is read from the rainfall chart. Overland flow and channel flow times are then recalculated to give a revised value of T_c. This iterative procedure is continued until the rainfall intensity i is adequately defined. Overland flow and channel flow times are determined as follows:

3a. Overland Flow Time

For small plots with relatively plane surfaces and without defined channels, such as highway pavements and medians, and built-up urban drainage areas, the Izzard[26] equation for time to equilibrium t_e in minutes is

$$t_e = \frac{41\ b\ L_o^{1/3}}{(Ci)^{2/3}} \tag{42}$$

where

$$b = \frac{.0007\ i + C_r}{S_o^{1/3}} \tag{43}$$

where

L_o = length of overland flow, ft
C = coefficient of runoff
i = design rainfall intensity, inch/hr
C_r = coefficient of retardance, see Table 14 below

TABLE 14

Values of C_r in Eq. (43)

Surface	Value of C_r
Smooth asphalt	.007
Concrete pavement	.012
Tar & gravel pavement	.017
Closely clipped sod	.046
Dense bluegrass turf	.060

Eq. (42) is valid when the product of iL_o is less than 500. Although such instances will be rare, where iL_o does exceed 500 Eqs. (44) and (45) may be used

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[26] Izzard, C. F., Hydraulics of Runoff from Developed Surfaces, Proc Highway Research Board, Vol 26, pg 129-150, 1946.

$$t_e = \frac{b' L_o^{.6}}{(Ci)^{.4}} \qquad (44)$$

where

$$b' = 1.2 \frac{n^{.6}}{S_o^{.3}} \qquad (45)$$

which are based on kinematic wave theory[27] and the Manning equation, with n the familiar Manning roughness coefficient.

If the drainage area has irregular surface topography rather than essentially flat surfaces, the Kirpich[8] equation previously referred to, is suggested as an alternate method of estimating t_e. This empirical relation,

$$t_e = \frac{.00013 L_o^{.77}}{S_o^{.385}} \qquad (46)$$

may be solved numerically or by the Kirpich nomograph in Appendix D. All terms are as previously defined.

The Kirpich equation does not include the rainfall intensity i, but assumes that t_e is constant for any given drainage area, irrespective of rainfall rate. A comparison of the Kirpich equation with other empirical equations for estimating t_e shows that it tends to give conservative results (i.e. Kirpich produces comparatively low t_e values).

3b. Channel Flow Time

If defined flow channels, such as gutters, ditches, or pipes, are included in the design, the designer must also estimate the travel time for flow in such channels. A common method of obtaining channel flow time, as suggested by Linsley and Franzini[28], is to divide the channel length L_c by the mean velocity of flow when the channel is at the bankfull stage. This method obviously is a simple approximation of the spatially varied channel flow phenomenon and should be used with caution if the drainage area exceeds 10 acres. To be conservative the designer may wish to use only about 2/3 of the value calculated in this manner. The Manning equation is suggested to estimate the mean channel velocity.

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[27] Eagleson, P. W., Dynamic Hydrology, McGraw-Hill Book Co., New York, NY 1970, p 340.

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{47}$$

where

V = mean velocity, ft/sec

n = Manning n

R = hydraulic radius = A/WP
(A = cross section area, WP = wetted perimeter)

S = slope energy grade line (for uniform flow
slope channel bed)

A comprehensive tabulation of n values, and two nomographs for graphical solution of the Manning equation are presented in Appendix D.

Izzard[29] also developed a method for estimating channel flow time. However his method appears rather cumbersome and apparently is not widely used.

4. With values of C, i, and A now determined we calculate the estimated flood peak from the rational formula as

$$Q = CiA \tag{40}$$

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[28] Linsley, R. K., J. B. Franzini, Water Resources Engineering, 2nd ed., McGraw-Hill (NY) pg 59.

[29] Izzard, C. F., Hydraulics of Surface Runoff from Developed Surfaces, Proc Highway Research Board, Vol 26, pg 147-150.

where

Q = Peak discharge, cfs

C = coefficient of runoff

i = Rainfall intensity corresponding to time of concentration of drainage area, inch/hr

A = Drainage area, acres

CHAPTER IV

CULVERT HYDRAULIC DESIGN

INTRODUCTION

This chapter covers the fundamentals of culvert hydraulics and presents recommended procedures for culvert selection. These procedures are based soley on the hydraulics of culvert flow and do not involve the economic aspects of culvert cost, installation, or maintenance. The hydraulic theory and design charts presented in this report are limited to conventional culverts which have a uniform barrel cross-section. These include circular, arch and oval culverts, constructed of corrugated metal or concrete and concrete box culverts. With regard to inlet construction, the discussion covers culverts with inlet projecting from roadway fill, culverts with inlet mitered to the embankment slope, and culverts with inlet headwalls, or wing walls. Much of the material presented is taken from HEC No. 5, U.S. Dept. of Transportation[30]. An introduction to improved inlet structures is also presented.

FLOW THROUGH CULVERTS

There are various methods of catagorizing flow in culverts. However for purposes of analyzing culvert performance and culvert selection it is essential to recognize (1) flow with inlet control and (2) flow with outlet control.

INLET CONTROL

Inlet control may occur either with the inlet unsubmerged or submerged as shown in Fig. 17A and B. When a culvert is flowing under inlet control the discharge capacity of the culvert is controlled by three factors associated with the entrance of the culvert. These factors are (1) the depth of the headwater, (2) the cross-sectional area of the culvert at the entrance, and (3) the type and shape of the inlet structure.

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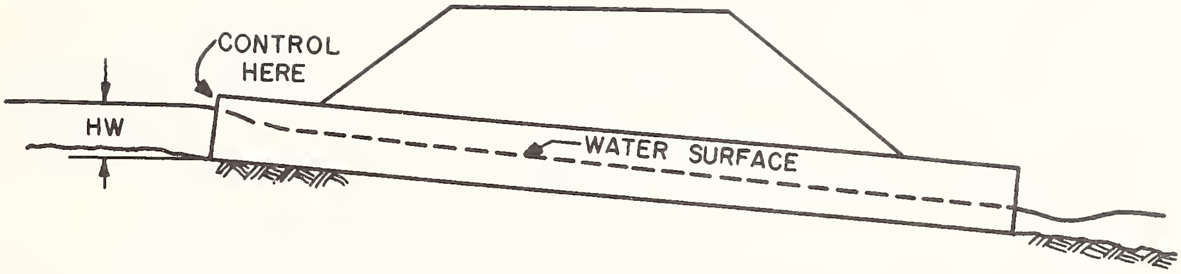
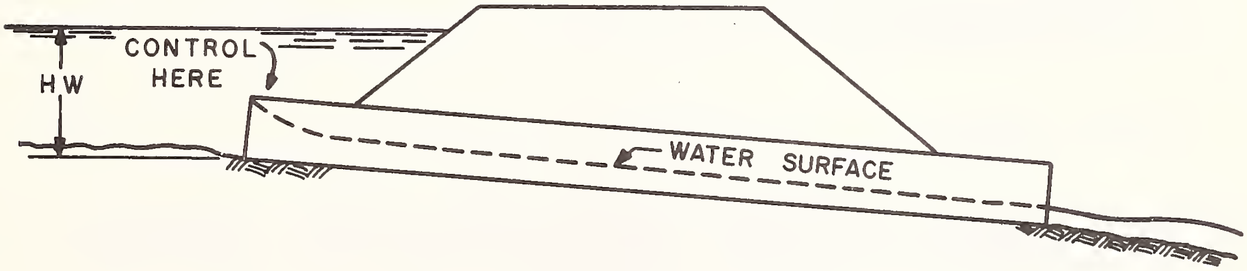
[30] Hydraulic Engineering Circular No. 5, U.S. Department of Transportation, Dec. 1965. Glossy Prints of Design charts 1-20 were provided by the Hydraulics Branch for reproduction in this report.



A. UNSUBMERGED INLET – INLET CONTROL



B. SUBMERGED INLET – INLET CONTROL

Fig. 17 Flow through Culverts with Inlet Control

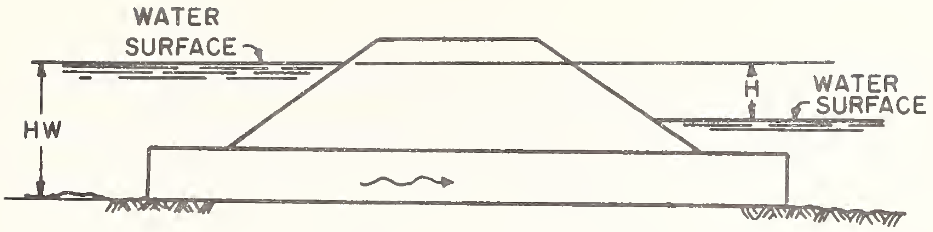
With unsubmerged inlet and inlet control (Fig. 17A) the flow either enters the culvert at a depth less than critical if flow in the approach channel is supercritical, or critical depth occurs near the inlet if flow in the approach channel is subcritical. In the latter case the flow is at a depth less than critical immediately downstream from the inlet in the culvert. See discussion under CRITICAL DEPTH.

With submerged inlet and inlet control flow (Fig. 17B) critical depth always occurs at or near the culvert inlet and the flow in the culvert immediately downstream is at a depth less than critical.

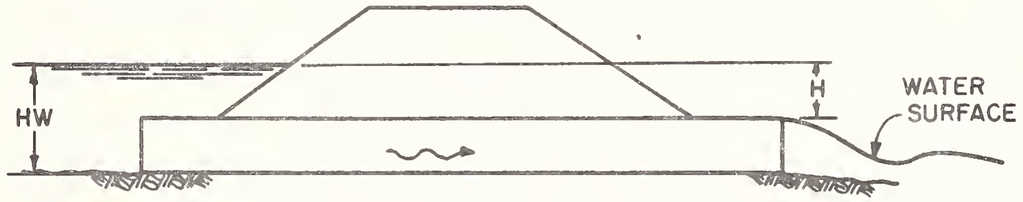
Thus inlet control always results in the occurrence of supercritical flow immediately downstream from the inlet. Furthermore, the flow will remain supercritical throughout the length of the culvert unless a hydraulic jump forms at some point in the barrel. The important point is that since flow just downstream from the inlet is always supercritical for inlet control, the headwater required to produce a given discharge through the culvert is not influenced by the downstream conditions. This means that the tailwater depth, or culvert barrel roughness and length have no influence on the headwater required to produce the discharge with inlet control flow conditions. While some experiments have indicated that slope changes may have a small influence on the headwater depth for a given discharge, this effect is usually considered negligible.

OUTLET CONTROL

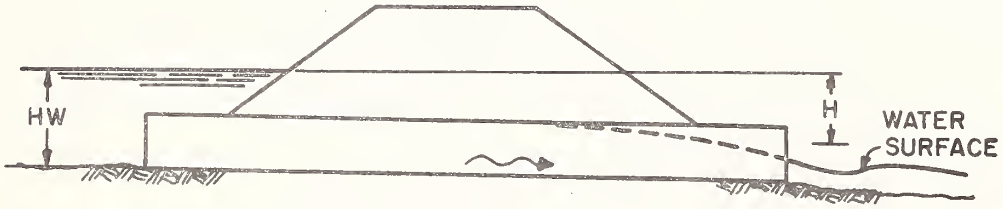
Culverts flowing with outlet control may flow with the barrel full or partially full for their entire length or with the barrel full for only a part of its length. Fig. 18A and B show the conditions for which a culvert with outlet control flows full through its entire length, while Fig. 18C and D show flow with outlet control for culverts with the barrel flowing partly fully for a portion of its length and for all the barrel length. The only difference between the inlet control culvert in Fig. 17A and the outlet control culvert in Fig. 18D is that the flow shown in Fig. 17A is supercritical while the flow in Fig. 18D is subcritical throughout the culvert. The procedure given in this report gives a reasonably accurate



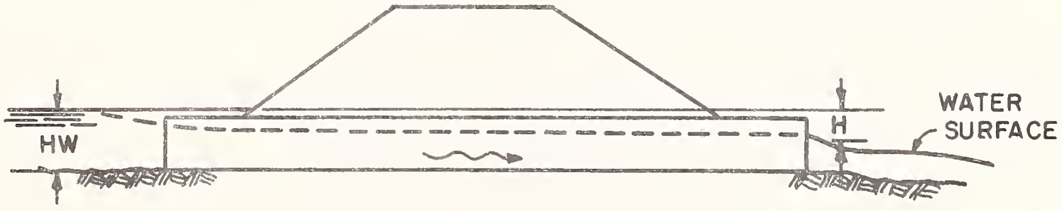
A. TAILWATER ABOVE CULVERT CROWN AT OUTLET



B. CULVERT JUST FULL AT OUTLET



C. CRITICAL DEPTH AND TW AT OUTLET BELOW CROWN—UPSTREAM PORTION CULVERT BARREL FLOWING FULL



D. CRITICAL DEPTH AND TW AT OUTLET BELOW CROWN—ENTIRE CULVERT FLOWING PARTIALLY FULL

Fig. 18 Flow through Culverts with Outlet Control

solution for required headwater for the type of flow shown in Fig. 18C. The part-full subcritical flow condition shown in Fig. 18D requires a backwater computation if accurate headwater depths are desired. The details of such computations are not covered here. Instead the procedure presented is the same as for the flow shown in Fig. 18C provided the headwater is .75D and above. For lower headwater the more detailed computations will be required.

The slope of the culvert barrel, in combination with its length, roughness, and the type of entrance determine whether the culvert will flow with inlet control or outlet control for a given discharge. For example, a short culvert laid on a mild slope may flow with inlet control even though the required headwater is above the inlet crown. On the other hand a culvert of sufficient length laid on a mild slope will flow with outlet control for the same discharge. It is possible by involved computations to determine the probable type of flow under which a culvert will operate for given conditions. However by computing the headwater depths required for the given discharge for both inlet control and outlet control from the charts presented in this report, and assuming the higher headwater value and flow condition to prevail, such computations may generally be avoided. For convenience all symbols used in the following discussion are defined in Table 15.

TAILWATER DETERMINATION

Tailwater depth TW is defined as the depth of the water in the natural channel at the outlet of the culvert, measured above the culvert invert. Depending on whether the culvert is flowing under inlet or outlet control, the depth of tailwater may be an important factor in determining the proper culvert size.

Where the downstream channel is well defined, TW may be calculated by the Manning equation or established from gage readings or high water marks. In either case the TW desired is that corresponding to the design discharge being used for culvert selection. The field survey should have determined whether tailwater depth is governed by some downstream obstruction or constriction in the channel. In many cases the channel downstream from the culvert has sufficient capacity that the tailwater depth is less than the

TABLE 15

Nomenclature for Culvert Design

Symbol	Units	
Q	cfs	Discharge or rate of flowing water in cubic feet per second; design discharge when written Q_{50} to indicate design frequency.
A	sq ft	Area of flow in square feet.
V	ft/sec	Mean velocity of flow (also v) in feet per second.
V_c	ft/sec	Critical velocity, or velocity at critical depth.
Y	ft	Depth of flow.
Y_c	ft	Critical depth, depth of flow at minimum energy. At depths greater than critical flow is subcritical, or tranquil. At depths less than critical flow is supercritical or turbulent.
Y_n	ft	Normal Depth of flow.
HW	ft	Headwater or depth of water a short distance upstream of the culvert inlet.
AHW	ft	Allowable headwater.
TW	ft	Tailwater or depth of water at or a short distance downstream of the culvert outlet.
h_o	ft	Control depth of water at the culvert outlet determined from the fraction $\frac{Yc+D}{2}$ or TW whichever is greater.
g	ft/sec^2	Acceleration of gravity = 32.2 feet per second squared.
H_V	ft	Velocity head = $\frac{V^2}{2g}$.
H_e	ft	Entrance head loss = $K_e \frac{V^2}{2g}$.
H_f	ft	Friction head loss due to pipe roughness.
K_e	--	Entrance loss coefficient (0.5 for aprons).
n	--	Manning's roughness coefficient.
D	ft	Diameter or rise of culvert.
B	ft	Width of box culvert.

depth of flow at the culvert outlet. In this case the tailwater does not serve as a control and need not be calculated accurately. A detailed example of tailwater computation is presented in Appendix E.

CRITICAL DEPTH

Critical depth, Y_c, defines an important parameter in open channel flow. It is of great physical significance in many practical problems.

When a given discharge, Q, flows in an open channel at such depth that the specific energy is a minimum, flow is said to occur at critical depth. Specific energy is defined by

$$E = Y + \frac{V^2}{2g} \tag{48}$$

The mean velocity at critical depth, $V_c = Q/A_c$ is called critical velocity. For any shape of cross-section critical depth occurs when the following relation is satisfied.

$$\frac{Q^2}{g} = \frac{A_c^{\;3}}{T_c} \tag{49}$$

where

Q = rate discharge, cfs
g = 32.2 ft/sec^2 (acceleration gravity)
A_c = cross-section area at critical depth, ft^2
T_c = water surface width at critical depth, ft

Since for a rectangular channel A = BY and T = B where B is width of channel bottom, Eq. (49) gives

$$\frac{Q^2}{g} = B^2 Y_c^{\;3} \tag{50}$$

or

$$Y_c = \sqrt[3]{\frac{q^2}{g}} \tag{51}$$

where q = Q/B, the discharge per ft width.

For other shapes of channel, although Eq. (49) is satisfied at critical depth, there are generally no simple formulas for calculating Y_c, such as Eq. (51). However a graphical solution illustrated by Fig. 19 may always be prepared for a defined geometric shape. Appendix E contains such critical depth charts for circular pipe up to 9 ft diameter, oval concrete pipe, and corrugated metal pipe-arch cross sections.

CHART 16

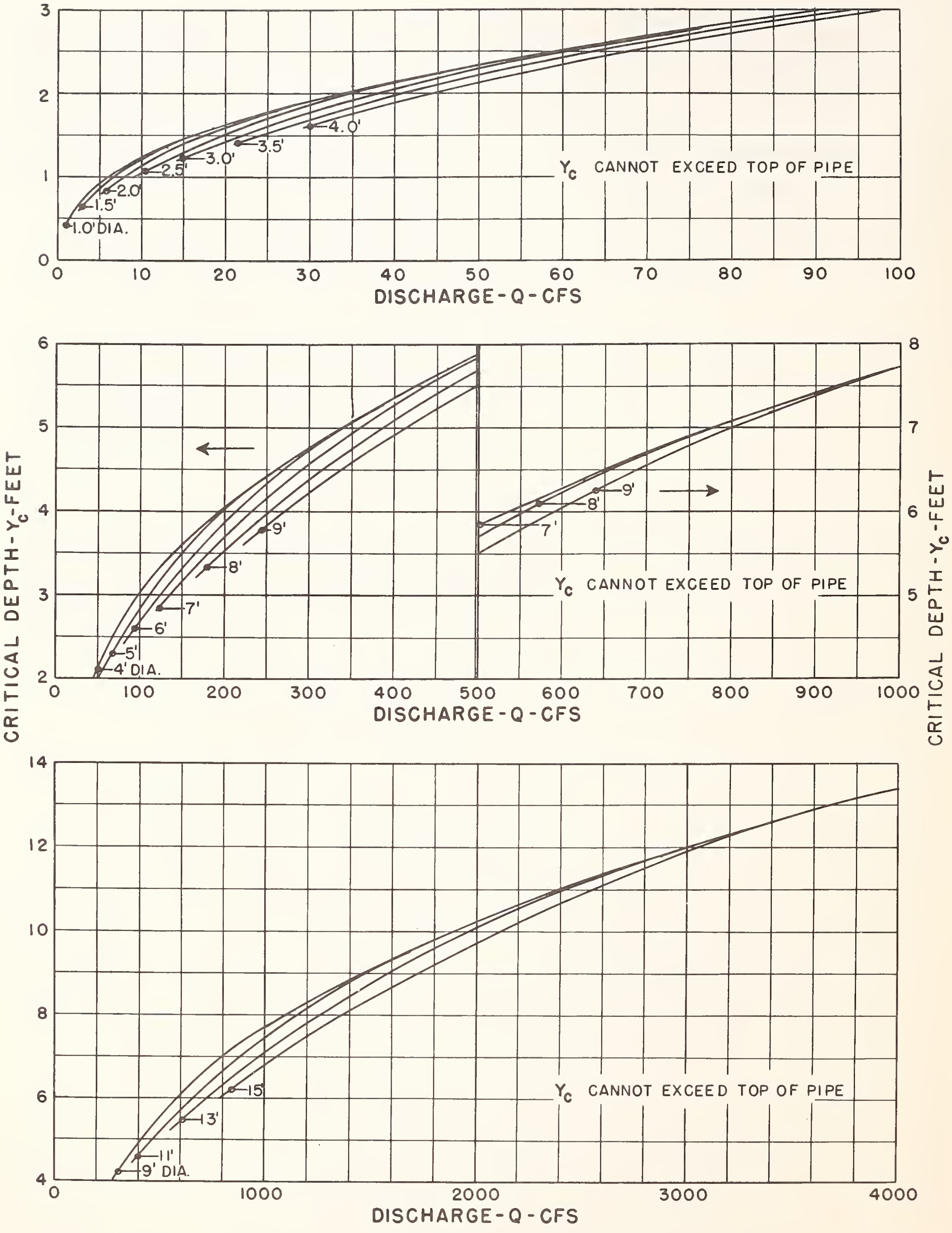


Fig. 19 Critical Depth for Circular Pipe

CULVERT SELECTION

Selection charts developed by the Federal Highway Administration for inlet control and outlet control conditions for the commonly used culvert types and sizes are presented in Appendix E. The inlet-control charts were based on experimental data. Any attempt to check the culvert capacity given by the charts for inlet control from basic theory require that assumptions be made for not only the energy loss at entrance, but also the contraction effect produced by the inlet geometry. For outlet control with culvert barrel flowing full, there should be good agreement between discharge read from the charts and that calculated from theory.

CULVERT SELECTION FOR INLET CONTROL

Fig. 20 shows an inlet control chart for circular corrugated metal culverts from 12 to 120 inch diameter. When the design discharge Q, type culvert and entrance, and size of culvert are specified the ratio of headwater to culvert diameter may be read directly from the nomograph.

CULVERT SELECTION FOR OUTLET CONTROL

When outlet control exists several other factors in addition to those considered for inlet control must be considered. Consideration must be given to the elevation of the tailwater in the channel at culvert outlet, the barrel length, slope, and roughness, as well as barrel shape and size and entrance geometry.

With outlet control the downstream water surface control elevation must be established. This control elevation, measured above culvert outlet invert, is designated h_o as shown in Fig. 21A and B.

These are four flow conditions which may occur.

(1) If the outlet of the culvert is submerged as in Fig. 21A, the control depth is equal to the tailwater depth, giving

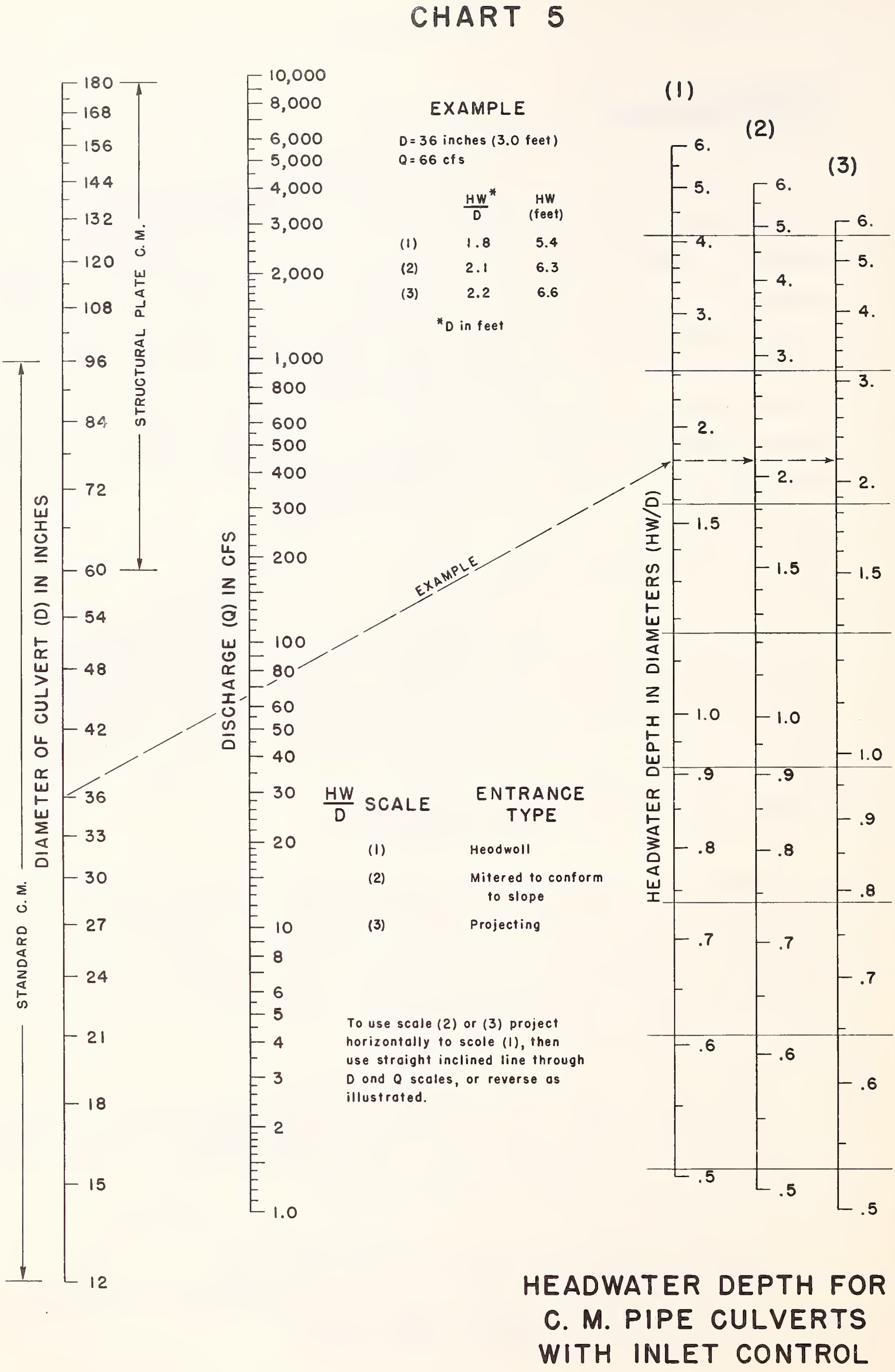


CHART 5

EXAMPLE

D = 36 inches (3.0 feet)
Q = 66 cfs

	$\frac{HW}{D}$*	HW (feet)
(1)	1.8	5.4
(2)	2.1	6.3
(3)	2.2	6.6

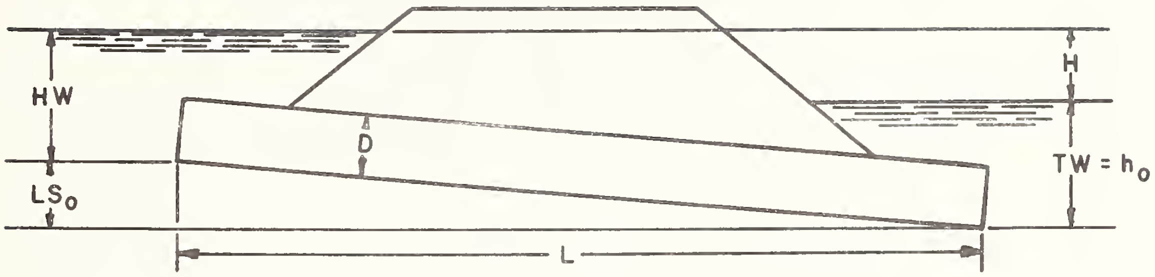
*D in feet

$\frac{HW}{D}$ SCALE	ENTRANCE TYPE
(1)	Headwall
(2)	Mitered to conform to slope
(3)	Projecting

To use scale (2) or (3) project horizontally to scale (1), then use straight inclined line through D and Q scales, or reverse as illustrated.

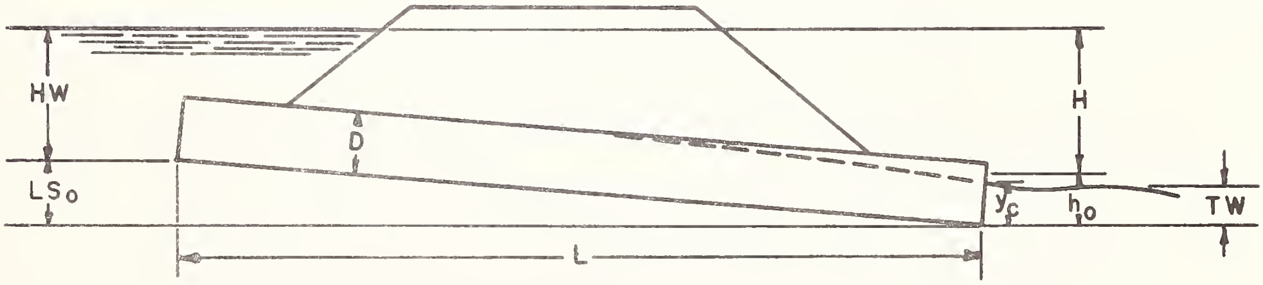
HEADWATER DEPTH FOR C. M. PIPE CULVERTS WITH INLET CONTROL

Fig. 20 Sample Design Chart for Inlet Control



$$HW = h_o + H - LS_o$$

A. OUTLET CONTROL — TW ⩾ CULVERT CROWN (D)



NOTE: WHEN TW $> \frac{y_c + D}{2}$ $h_o = TW$

WHEN TW $< \frac{y_c + D}{2}$ $h_o = \frac{y_c + D}{2}$

B. OUTLET CONTROL — CRITICAL DEPTH AND TW AT OUTLET BELOW CROWN (D)

Fig. 21 Control Depth for Outlet Control Flow

$$h_o = TW \tag{52}$$

(2) If the culvert is flowing completely full with the outlet unsubmerged as in Fig. 18B, previously shown, the control depth is equal to the diameter of the culvert. This condition exists for an outlet control culvert when the critical depth Y_c exceeds the culvert height D. Then

$$h_o = D \tag{53}$$

(3) The culvert may flow full for only a portion of the barrel as shown in Fig. 21B. For this flow condition to exist both critical depth Y_c and tailwater TW must be less than D. Fig. 22 shows this flow condition in more detail. Here the hydraulic grade line will pass through the point A where the water surface breaks with the top of the culvert.

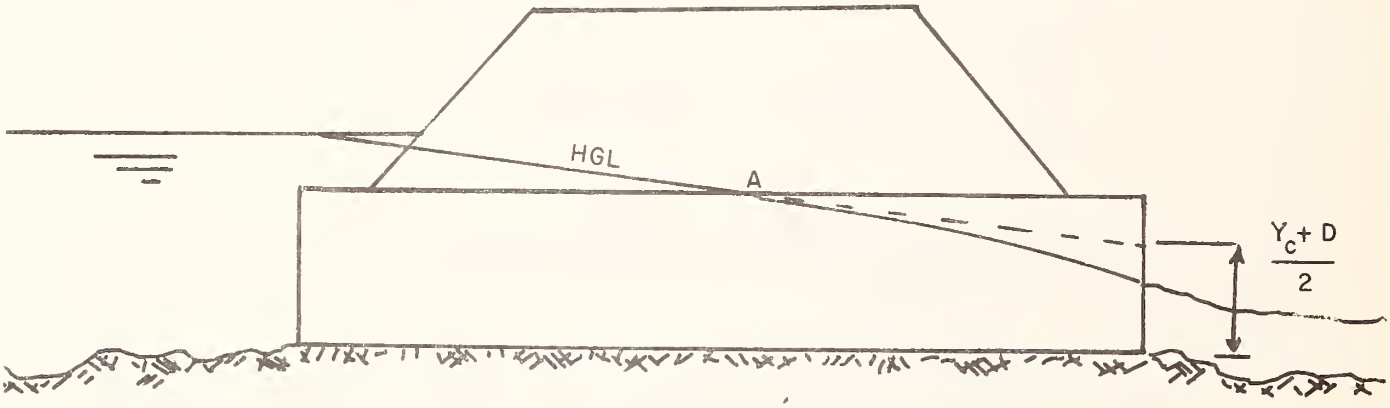


Fig. 22 Approximate Hydraulic Grade Line at Culvert Exit

Detailed computations show that the hydraulic grade line, if extended through A as a straight line, intersects the plane of the outlet cross-section at a point above critical depth. The location of this point is approximately midway between critical depth and the crown of the culvert. Consequently we may approximate h_o for this flow condition as $h_o = (Y_c + D)/2$, provided TW is less than this value. The following relations then apply:

$$h_o = TW \qquad \text{if } TW > \frac{Y_c + D}{2}$$

$$h_o = \frac{Y_c + D}{2} \qquad \text{if } TW < \frac{Y_c + D}{2} \tag{54}$$

(4) Flow may occur as shown in Fig. 18D, when the culvert barrel flows partially full throughout its length. As previously stated, the accurate determination of headwater for this condition would require a backwater computation. However if HW exceeds .75D, experience has shown that the outlet control depth h_o may be approximated as in (3) above.

For flow condition (3) and flow condition (4) (provided HW > .75D) above, we use the full flow culvert charts for determination of H. Actually the velocity is somewhat higher along the partially full portion of the culvert than in the full-flow portion. However, if the velocity in the partial full region is approximated by the value obtained for full pipe flow, it then becomes easy to calculate friction loss in the culvert barrel. An involved non-uniform flow computation would otherwise be required. For flow condition (3) the method should be a good approximation to the more lengthy alternate non-uniform flow hydraulic computations. The estimated value of h_o is somewhat high, which tends to compensate for the assumption of a low velocity in the partial full region. For flow condition (4) the method is the less accurate and should not be relied upon if HW is less than .75D.

When a culvert is flowing under outlet control flow conditions the head H is the total head required to discharge the design flow measured from the control depth h_o to the elevation of headwater. See Fig. 18.

Assuming full flow throughout the barrel as shown in Fig. 18A and B, H from the energy equation is the sum of the velocity head H_v, an entrance loss H_e, and a barrel friction loss H_f, or:

$$H = H_v + H_e + H_f \tag{55}$$

Velocity head $H_v = V^2/2g$, where V is the average velocity in the culvert barrel. Each term in Eq. (55) is expressed in units of energy per pound of fluid ft-lb/lb, or dimensionally ft. The velocity head of the water in the approach channel is neglected in the procedures given in this report.

The entrance loss H_e represents the head loss as the water passes through the entrance of the culvert. The amount of entrance loss depends on the type of entrance structure used and is expressed as

$$H_e = K_e \frac{V^2}{2g} \qquad (56)$$

The barrel loss H_f is the head loss between the entrance and outlet of the culvert which varies directly as the length of culvert barrel. There are several ways of expressing H_f, but most highway engineers are familiar with the Manning formula Eq. (47), from which the following expression is derived

$$H_f = \frac{29n^2L}{R^{1.33}} \quad \frac{V^2}{2g} \qquad (57)$$

where

H_f = Head loss, ft

n = Manning n

R = hydraulic radius of culvert, ft

L = Length of culvert, ft

$\frac{V^2}{2g}$ = Velocity head, ft

Note: In the conventional form of the Manning formula

$S = H_f/L$

Values of K_e for various types of culvert entrance are given in Table 16 and n values for conventional culvert materials are presented in Appendix E.

With values of K_e and n known, one reads values of H for the design discharge and for various culvert types and sizes directly from the selection charts for outlet control which are presented in Appendix E. Fig. 23 which

TABLE 16

Entrance Loss Coefficients

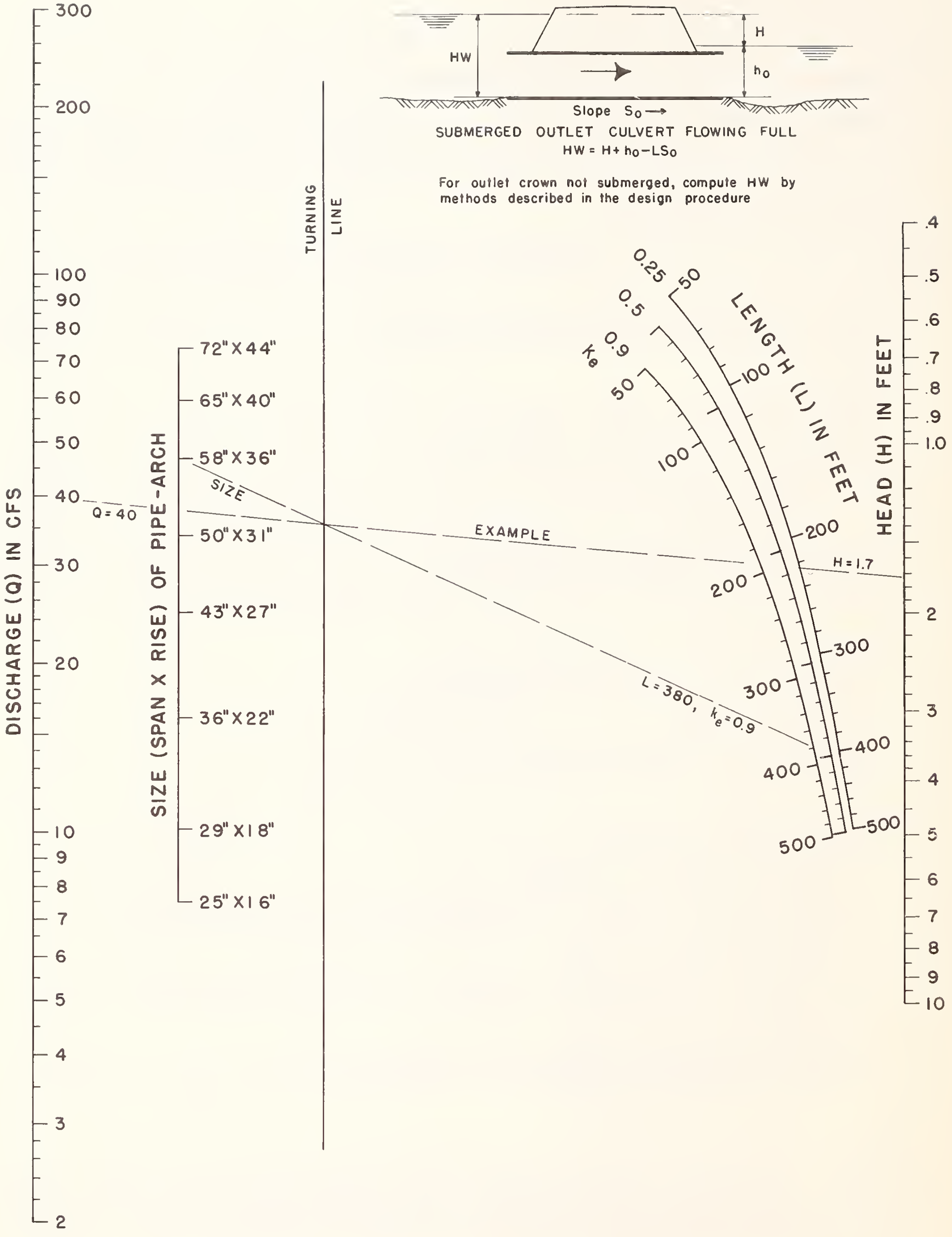
Coefficient K_e to apply to velocity head $\frac{V^2}{2g}$ to determine loss of head at entrance to a structure, such as a culvert or conduit, operating full or partly full with control at the outlet.

$$\text{Entrance head loss } H_e = K_e \frac{V^2}{2g}$$

Type of Structure and Design of Entrance	Coefficient K_e
Pipe, Concrete	
Projecting from fill, socket end	0.2
Projecting from fill, sq cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe	0.2
Square edge	0.5
Rounded (radius = 1/12 D)	0.2
Mitered to conform to fill slope	0.7
*End-Section conforming to fill slope (Apron)	0.5
Pipe, or Pipe-Arch, Corrugated Metal	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls	
Square-edge	0.5
Mitered to conform to fill slope	0.7
*End-section conforming to fill slope (Apron)	0.5
Box, Reinforced Concrete	
Headwall parallel to embankment	
Square-edged on 3 edges	0.5
Rounded on 3 edges to radius of 1/12 barrel dimension	0.2
Wingwalls at 30° to 75° to barrel	
Square-edge at crown	0.4
Crown edge rounded to radius of 1/12 barrel dimension	0.2
Wingwalls at 10° to 30° barrel	
Square-edge at crown	0.5
Wingwalls parallel (extension of sides)	
Square-edged at crown	0.7

*Note: "End Section conforming to fill slope," made of either metal or concrete, are the sections commonly available from manufacturers. From limited hydraulic tests they are equivalent in operation to a headwall in both inlet and outlet control.

CHART 12



HEAD FOR
STANDARD C. M. PIPE-ARCH CULVERTS
FLOWING FULL
n=0.024

Fig. 23 Sample Design Chart for Outlet Control

was prepared for standard corrugated metal culverts from 12 to 180 inch diameter illustrates these charts. Each chart was prepared for a particular value of n and for several K_e values. However the chart may be used for different n and K_e values as described in Appendix E.

Finding a value of H from a nomograph does not complete the solution in the case outlet control flow. It is also necessary to take into account the slope and length of the culvert along with the control depth h_o. Thus from the energy equation the headwater depth HW is determined as

$$HW = H + h_o - LS_o \tag{58}$$

See Figs. 18A and B.

VELOCITY FLOW AT CULVERT OUTLET

The high velocities created in a culvert may cause serious erosion problems at the culvert outlet or immediately downstream from the culvert. Such erosion may cause intolerable degradation of the downstream channel and/or costly and dangerous embankment failures. Because of this, the outlet velocity should always be a factor in culvert design.

The outlet velocity of a culvert flowing with inlet-control should be taken as the larger of the values calculated from the assumptions of normal depth or critical depth at outlet. The velocity at normal depth is determined using the Manning equation, previously defined as

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{47}$$

Recall that for inlet-control we assume that critical depth occurs near the entrance. If the normal depth Y_n is less than the critical depth Y_c, that is if the culvert slope is hydraulically steep, then the velocity at normal depth will always be higher than the critical velocity and thus will be a conservative estimate. However if Y_n is greater than Y_c, that is if the culvert slope is hydraulically mild, the formation of a hydraulic jump would be required to cause normal depth at culvert outlet to occur with inlet control flow. It is entirely possible however that a hydraulic jump will not form in an inlet controlled culvert with a mild slope. If the hydraulic jump does not occur, the velocity at the outlet will be supercritical instead of subcritical as predicted by the Manning Equation. Therefore the normal velocity will be considerably lower than the actual outlet velocity. To prevent the prediction of a low outlet velocity by the Manning Equation (assuming that a detailed water surface profile is not determined) the velocity should also be calculated assuming that critical depth exists at the outlet, and the larger velocity assumed to occur.

Critical velocity may be calculated in different ways. However, if it is known that the flow is critical, A_c the cross section area at critical depth may be determined and V_c calculated from

$$V_c = \frac{Q}{A_c} \tag{59}$$

Detailed procedures for determining critical depth have been presented earlier in this chapter.

For outlet control flow the average velocity at culvert outlet may be determined by dividing the discharge by the cross-sectional area of flow at the outlet. This area will have been defined in establishing the control depth at outlet, as critical depth, tailwater depth, or the full cross section of the culvert barrel.

PREVENTION EROSION AT CULVERT OUTLET

In general a culvert always increases the velocity over that which would have occurred in the natural channel. A potential erosion problem is therefore created at each culvert installation. When the outlet velocity is too high adequate channel and embankment protection is required. The judgment of experienced local engineers is required to determine the extent of the protection required. In severe cases energy dissipation structures will be required. Where local conditions permit channel and embankment may be adequately protected by properly designed and placed rip-rap.

Some of the local conditions which determine the type of protection to be provided are (1) erodibility characteristics of soil in the stream channel, (2) the predicted culvert outlet velocity, (3) the maximum velocity to be expected in the stream channel (in many cases this is higher than the average velocity), (4) danger to traffic and human life, and (5) damage to crops, structures, and highway if erosion does occur.

It is not the intent of this report to specify guidelines for erosion control. It is strongly recommended, however that outlet conditions at the culvert be reviewed by a competent local engineer who is familiar with the conditions which exist and the erosion history in the immediate vicinity of the culvert site.

IMPROVED INLET STRUCTURES

The Federal Highway Administration has published an investigation of the benefits to be derived from inlet structures designed to improve the capacity of culverts in HEC 13, Hydraulic Design of Improved Inlets for Culverts, 1971. It is evident from this study that the possible economic and hydraulic merits of improved inlet design warrant consideration as a part of the culvert design procedure. However it is not the intent of this report to cover the many aspects of improved inlet design as these are already covered in detail in HEC 13. Rather, an attempt is made here to introduce the reader to the advantages and disadvantages of improved inlet design and also to help the designer decide under which circumstances he should consider the use of an improved inlet. The designer is referred to HEC 13 for the actual improved inlet selection procedure.

TYPES OF IMPROVED INLETS FOR CULVERTS

The research conducted by the FHWA resulted in the adoption of six different types of improved inlet structures, each having a particular application potential. Sketches of each type, including details, are shown in Figs. 24-27. Descriptions of each type are given in HEC 13, pgs. 19-35. Types I, II, III, IV apply to concrete box culverts while Types V & VI apply to circular culverts. It should be emphasized that the simplest type of inlet improvement may be the beveled edge inlet, if this has not already been incorporated in the design. It has been found that the change of conventional culverts with square-edge inlets to beveled-edge inlets will increase the capacity of inlet controlled culverts by 5 to 20 percent. Because of this the FHWA recommends that bevels be used on all culverts operating under inlet control. In fact, since little additional cost is involved, it is recommended that bevels be included on all culvert entrances, even though the capacity will not be increased as much for outlet control. Effective bevels are achieved with circular concrete pipe by simply using the socket end as the inlet. For concrete box culverts a bevel as described in HEC 13 may be easily formed at the entrance edge. For corrugated metal pipes of various shapes a suitable bevel may be used if a concrete headwall is incorporated in the design.

ADVANTAGES AND DISADVANTAGES IMPROVED INLET STRUCTURES

Increased Capacity Existing Culverts

The hydraulic capacity of existing culverts flowing with inlet control may be significantly increased merely by the installation of an improved inlet structure. Fig. 28 shows performance curves of different types of inlet structures for a 6 ft x 6 ft reinforced concrete box (RCB) culvert 200 ft long, layed on a 2% grade (4 ft drop in elevation). For flow with inlet-

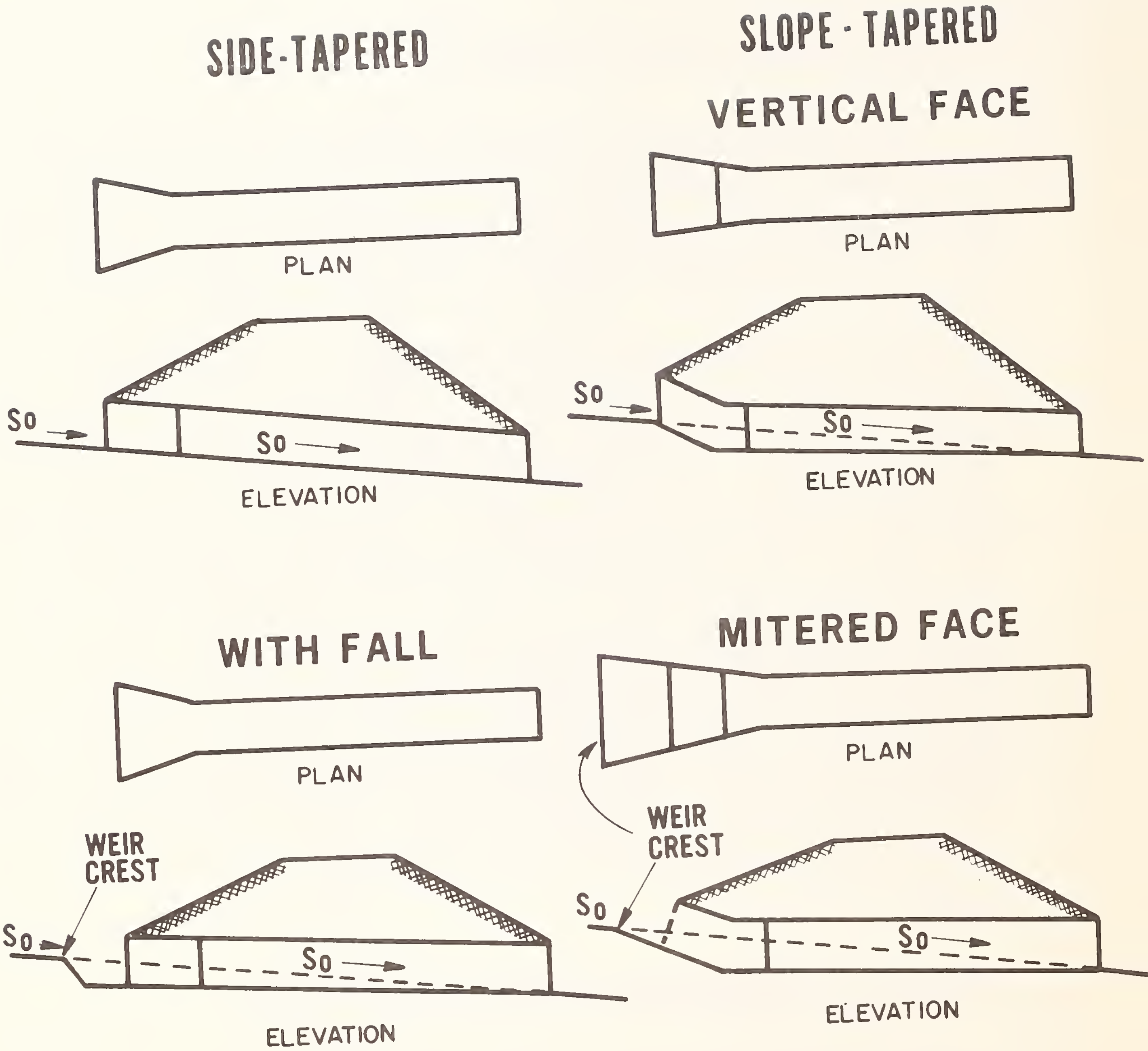


Fig. 24 Types of Improved Inlets for Box Culverts

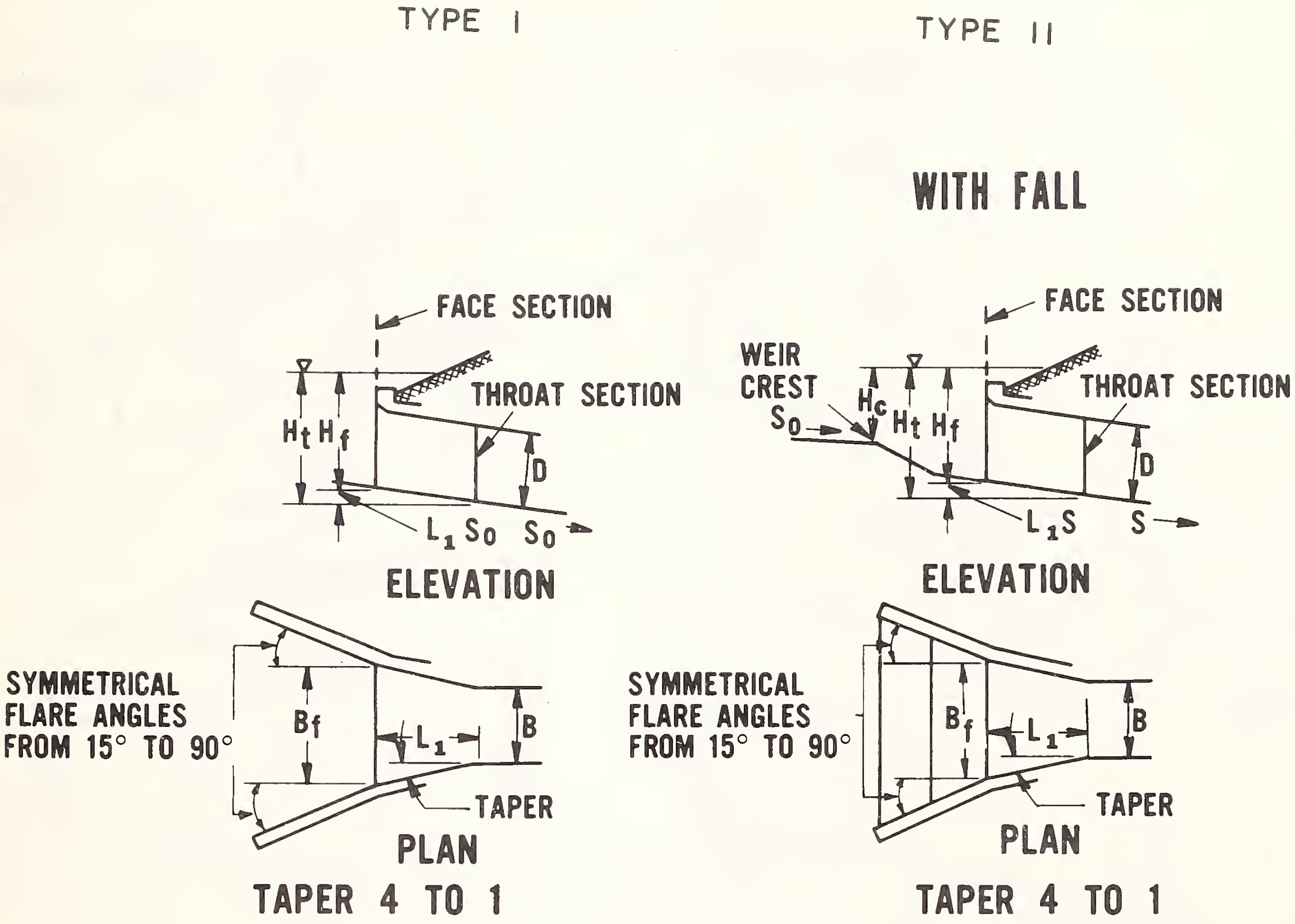


Fig. 25 Details of Improved Inlet Types I and II

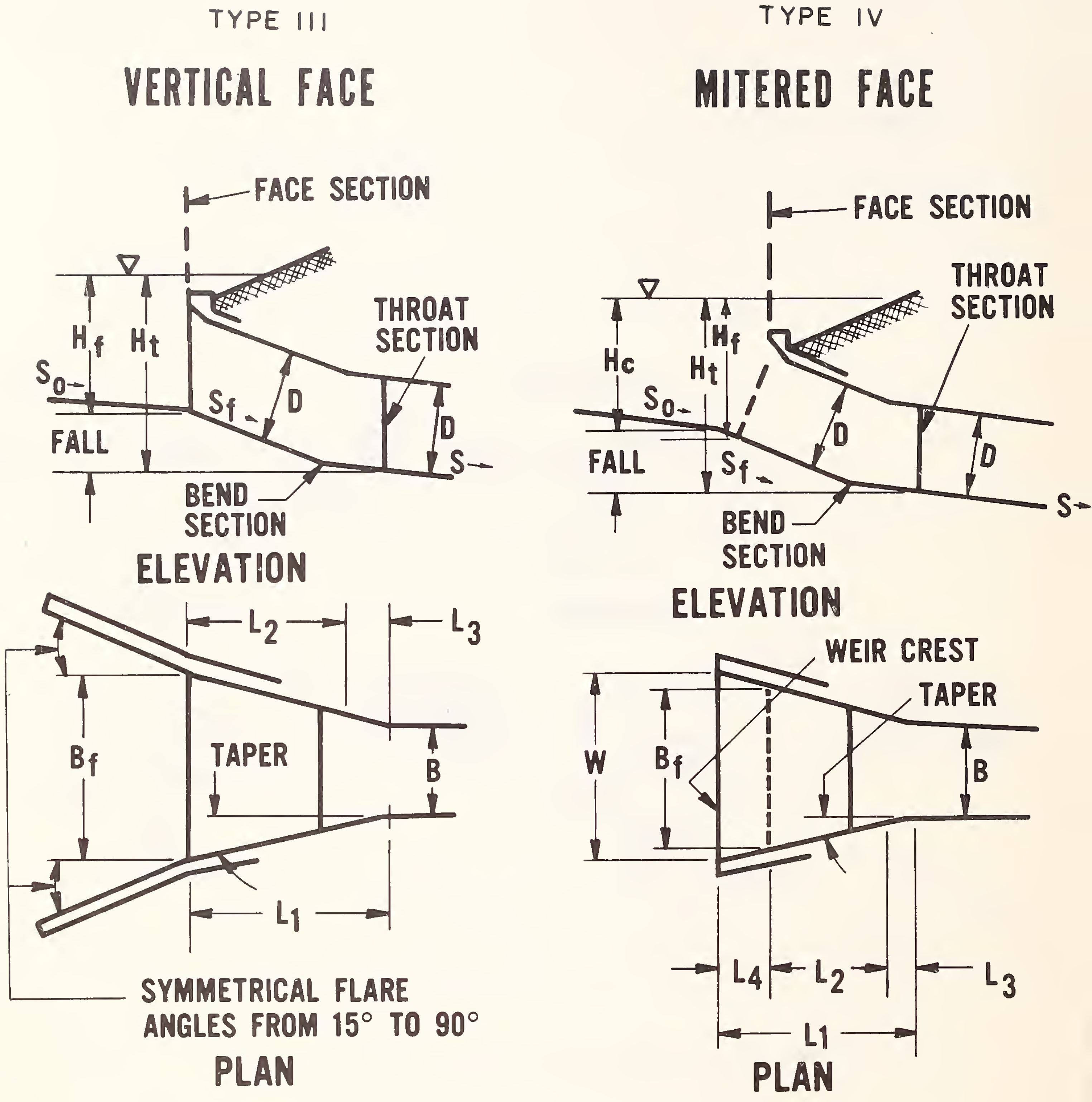


Fig. 26 Details of Improved Inlet Types III and IV

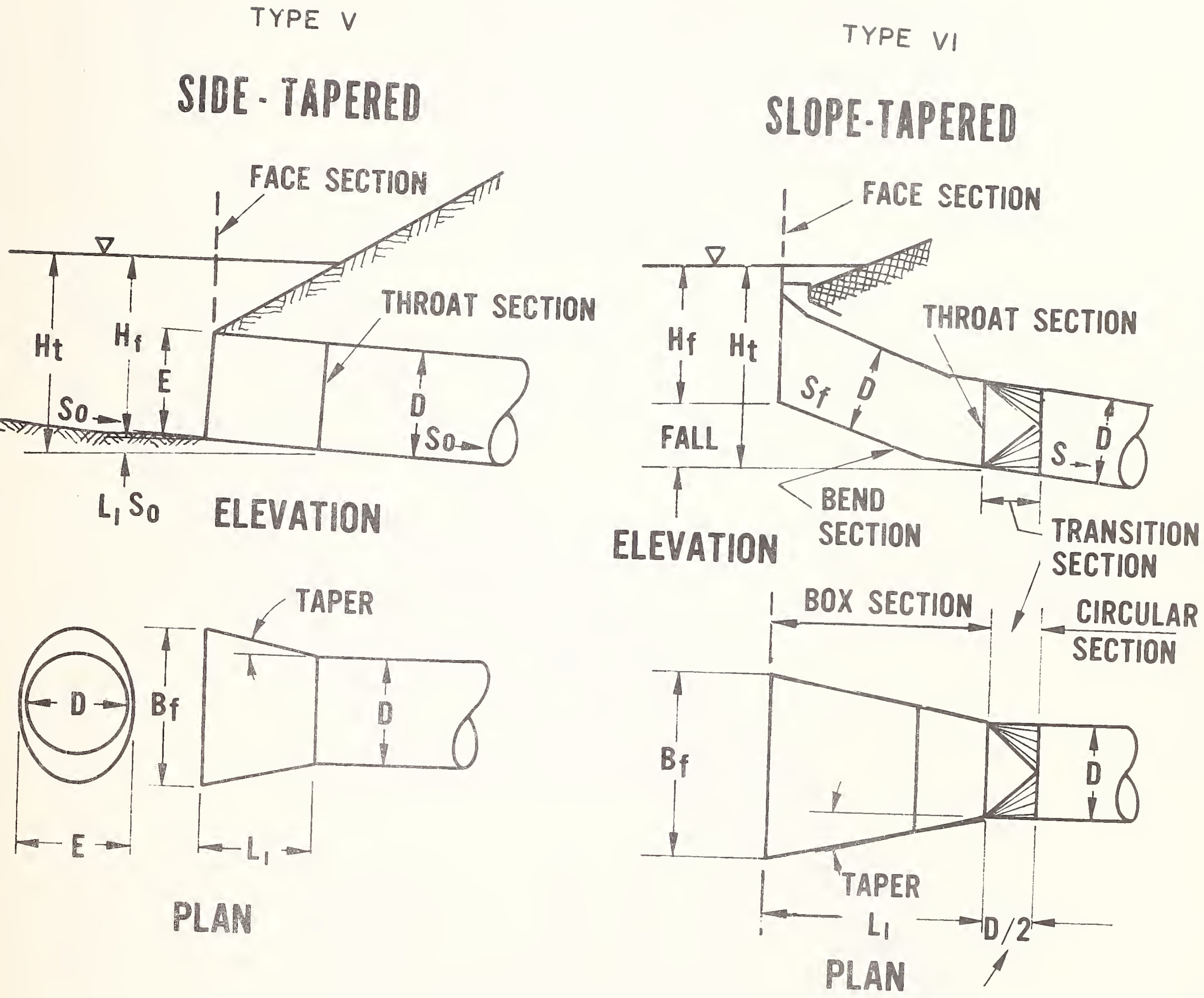


Fig. 27 Types of Improved Inlets for Pipe Culverts

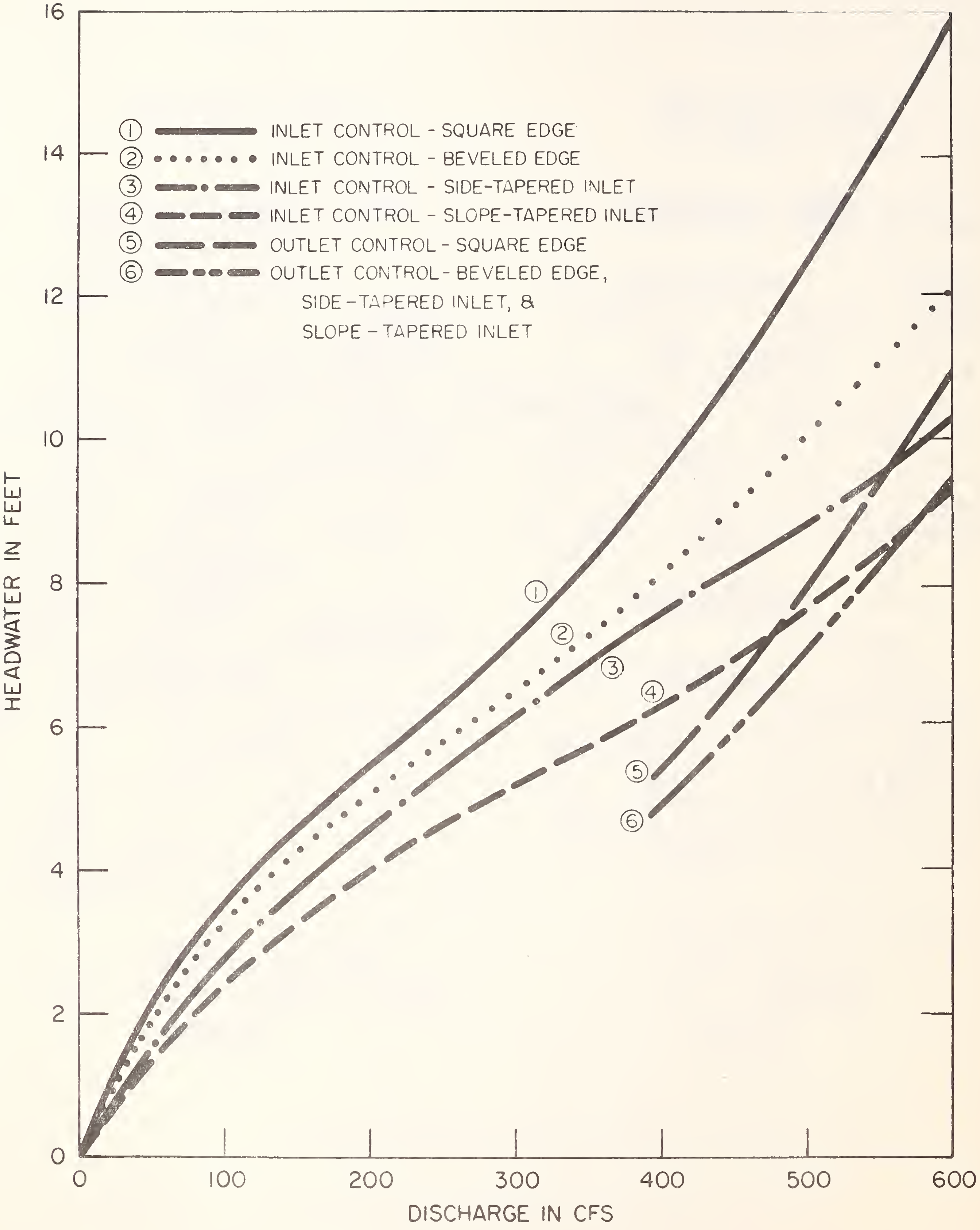


Fig. 28 Performance Curves for Single 6' x 6' Box Culvert with 90° Wingwall

control, it can be seen from curves 1-4 that the headwater required to pass a given discharge may be reduced considerably from that required for a square-edge inlet by using the modifications shown. It is recommended that this alternative be checked thoroughly in instances where existing culverts are too small. If it is found that an improved inlet structure will increase the culvert capacity to the desired level, it is quite likely that the modification of the inlet by incorporating a new inlet design will be much more economical than the replacement of the existing culvert.

Cost Savings in Culvert Design due to Reduced Culvert Size

It may be possible, through the use of an improved inlet structure, to pass the design discharge through a smaller culvert than would otherwise be required. This is because the net effect of an improved inlet is to reduce the amount of the entrance loss thus reducing the amount of headwater necessary to pass a given discharge.

In the FHWA study the costs of a total of 77 improved inlet culverts installed in 40 states were compared with the estimated costs for conventional culverts. The average percent savings realized for each type of improved inlet structure are presented in Table 17.

TABLE 17

Estimated Cost Savings Improved-Inlet
Over Conventional Culverts 40 States by 1971

Type Inlet	I	II	III	IV	V	VI
Number Each Type Installed	26	0	37	3	10	1
Average Percent Savings	29%	*	29%	*	22%	*

*Information insufficient to calculate meaningful average.

It is emphasized however, that improved inlets are much less significant in increasing the capacity of outlet control culverts than for inlet control culverts. Thus if the HW required for a given culvert design is based on outlet control, rather than inlet control, it is not recommended that a relatively expensive inlet structure be considered.

COST OF IMPROVED INLET CULVERTS

From the preceding discussion we see that the costs involved in the design and construction of improved inlets may or may not be greater than with conventional culverts. Because of this it is necessary for the designer to make a detailed cost analysis of all possible alternatives when considering the use of improved inlets designs. This process can become considerably more involved than the design procedure for conventional culverts. Realistic cost estimates for Type II, III, IV, & VI entrance structures will allow for the increased hand excavation required to provide the proper entrance channel and bedding needed for these inlet structures. Furthermore all types of improved inlet structures shown require additional man-hours to design, detail and build. The designer will need to be familiar with all the economic aspects in order to make the most intelligent decision.

The FHWA study also presented actual cost data for the 77 improved inlet culverts which had been installed by 1971. The costs are presented in Table 18.

TABLE 18

Cost of Improved Inlet Structures

	TYPE I	TYPE II	TYPE III	TYPE IV	TYPE V	TYPE VI
Number in use	26	0	37	3	10	1
Percent of Total (77)	34%	0	48%	4%	13%	1%
Average cost/sq ft x lineal ft	$1.90	*	$2.20	*	$1.80	*

*Information insufficient to permit calculation of a meaningful average value

The average cost in dollars per square ft of cross-section area per lineal ft of culvert applies only to the improved Inlet Type under which the value is tabulated and of course is an average among states for the year 1971. These cost values were obtained by taking a numerical average of the individual cost per unit area per unit length values. Since the square ft of culvert barrel required when these Improved-Inlets are used is apt to be considerably less than that required for a standard culvert inlet, no comparison should be made between the cost figures in Table 18 and comparable values for culverts with standard inlets. Instead the total cost of the culverts, rather than cost per sq ft barrel area must be compared to determine which is more economical.

The cost data presented in Table 18 is therefore only presented to give the designer a method of estimating the approximate in-place cost for a culvert with a particular type improved inlet after the designer had determined the size culvert barrel required for the type inlet being used. The units of these cost index values, which are \$/sq ft of barrel per linear ft of culvert allow a rough cost estimate to be made by multiplying the unit cost given by the barrel area and length of the proposed culvert. For example the estimated cost of 5 ft x 5 ft reinforced concrete box culvert 200 ft long with a Type I inlet would be found as

Cost = \$1.90 x 25 x 200 = \$9520

These costs very likely do not apply in Montana, as they were obtained using cost data collected in all parts of the United States. However the designer may find this information useful in making the initial decision as to whether or not an improved inlet should be considered at a particular site.

SELECTING IMPROVED INLETS FOR CULVERTS

It is assumed that the designer has selected a tentative conventional inlet culvert for which inlet control flow has governed.

Refer to HEC 13, pg. 50-53, for design Charts 1-3.

The first type of improved inlet that should be considered is the beveled-edge. For rectangular box culverts flowing with inlet control, the HW/D ratio can be obtained for two suggested standard designs of beveled edges (1:1 and 1.5:1) from Chart 1. This chart allows a quick comparison to be made between the HW/D ratio for a square edged entrance and for a beveled edge entrance. For similar analysis of beveled edge entrances for circular culverts the designer is referred to Chart 7, Appendix E.

If it is desirable to analyze the more sophisticated inlet control structures, the designer is referred to HEC 13, pgs 13-43. Here a stepwise procedure is presented for determining the hydraulic feasibility of each type of improved inlet. The selection of improved inlet-structures for circular culverts is very similar to that given for box culverts and is covered in the example problems in HEC 13 (Appendix A, pg 13-107 of that report).

It should be emphasized that the total design procedure for improved inlets involves more than just sizing the culvert to safely pass the design flow. In addition, the designer should conduct a thorough cost study to determine if the overall cost of the improved inlet structure is appreciably less than the cost of a conventional culvert. Only then will he want to adopt the improved-inlet design.

Types of costs for an improved inlet-structure for culverts (with the exception of the beveled edge) which need to be considered and which are not involved with a conventional culvert are:

1. Engineering costs associated with the design and economic analysis of the improved inlet.
2. Increased excavation costs for Improved Inlet designs.
3. Labor and material costs for the more involved inlet structure.
4. Any additional back fill and tamping costs associated with the geometry of the improved-inlet structure.

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